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THE ART OF GENERATING GEAR-TEETH.

BY

HOWARD A. COOMBS.

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The Art of Generating Gear Teeth.

CHAPTER I.

THEORY OF TOOTH-CURVES.

The theory of the action of gear teeth has probably received more attention by writers on mechanism than any other branch of applied mechanics, but comparatively little has been written with the intention of showing what has been done in the way of practically incorporating the principles involved in operative machines. Usually in works on the subject or in articles of the technical press not more than one or two machines are described, and there has not hitherto been published, as far as known to the writer, any comprehensive review of what has hitherto been accomplished in

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the invention and actual construction of gear-cutting machines. It is not proposed, however, to treat here of gear-cutting broadly, but to confine this review to those classes of gear-cutting machines which "generate" the tooth-curves, in contradistinction to those machines in which formed cutters or templets are used, previously shaped to the contour of the teeth. It is true here, however, as it is everywhere in nature as well as in mechanics, that a hard and fast line cannot be drawn between the classes.

Before describing any of the different ways in which inventors have solved the problems involved in the design and construction of gear generating machines, it will be advisable to state briefly what the problems are—that is, to show what the machines must be capable of doing before describing the ways in which they do it. Notwithstanding the fact that so much has been written about the theory of gear teeth, it is proposed to devote a little space to it here because

some familiarity with it is essential to an understanding of the principle of operation of generating machines, and it would not be safe to assume that everyone who will read this article will have the matter fresh in mind.

Instead, however, of approaching the subject from a purely mathematical point of view, as is usually the case in works on the subject, it is intended to present some special concrete cases in such a way that the conclusions arrived at can be *seen* to be true, with little or no mathematical demonstration. The practical man is usually satisfied if he can clearly see the application of the theory to some special case, and is willing to take for granted that the same holds true in all other cases of the same kind, but if he should not be satisfied without a general demonstration there are always the standard books on mechanics and treatises on toothed wheels to which he can refer.

Then, too, there are many people who

are deterred by a fear that they will not be able to follow from reading anything containing mathematical symbols, but there is no reason why the principles determining the forms of gear teeth in common use should not be understood by anyone of intelligence, even if he has never studied algebra or trigonometry. There is no branch of mechanics in which the application of theory to practice is more direct and beautiful than that which treats of the problem of giving the most efficient form to the teeth of gears.

According to the plan of taking a simple concrete case instead of a general proposition covering different forms of gears, suppose we have a pair of parallel shafts which we wish to connect by a pair of spur gears in such a way that their relative motions will be exactly the same as if they were connected by a pair of smooth pulleys touching each other along a line and rolling on each other without any slip. The problem in designing gear teeth is simply to keep the

relative motions of a pair of gears the same as that of the smooth wheels. In the case under consideration the action of the teeth can be perfectly investigated on paper, since a section at right angles to the faces of the teeth is a plane. Thus in Fig. 1 the circles around O and A represent an end elevation or cross-section of a pair of smooth pulleys, which touch each other at C. If one of these is turned at a constant speed the other will be turned in the opposite direction also at a constant speed, which, assuming as we do that there is no slip, is the highest speed that the first pulley is capable of directly imparting. Obviously the circumferences are travelling at the same rate, since one is "pulled along" by the other without any lost motion, but as the circles have different radii the smaller will make a complete revolution before the larger one does; in other words, their angular velocities are not the same, the relation between them depending upon the ratio between the

radii. Since the angular velocity of the small circle is greater than that of the

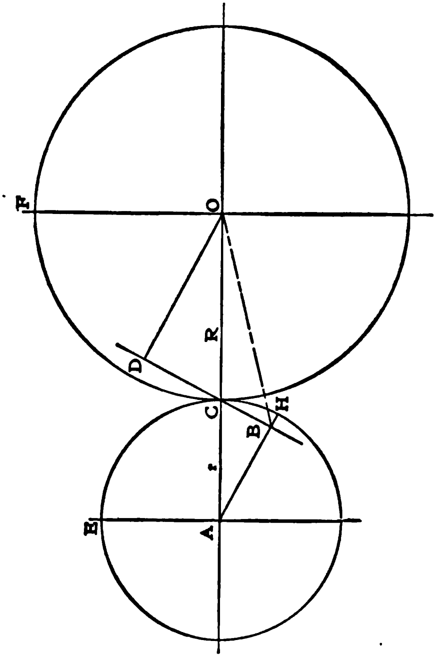


FIG. 1.—Principles of Gear-teeth Curves.

large one, we can say that the angular velocities vary inversely as the radii—

that is, the shorter the radius the greater the angular velocity, instances of which fact are constantly met with in machinery where a large pulley or gear drives a small one the shaft of which runs at a much higher speed than the shaft of the large wheel.

Now, as soon as a pair of toothed wheels takes the place of the smooth pulleys, the mutual action is no longer confined to the point C, but continues from the point where a pair of teeth first touch, through the point C, until they separate on the other side. Considering a single pair of teeth, it will be evident that when their point of contact crosses the line of centers AO it must do so at C, otherwise the ratio between the angular velocities of the two toothed wheels would not be the same as that between the angular velocities of the two smooth wheels which touch each other at C only. Now, unless different parts of the same wheel are to be driven at different angular velocities, which is absurd and mani-

festly impossible, all the successive points of contact of the teeth from the time they first meet on one side of the line of centers until they separate on the other side must be so located that the same angular velocity will be transmitted as when the point of contact is at C, notwithstanding the fact that the driving point is at a greater distance from the center, and consequently is travelling faster. This appears like a serious obstacle until one considers that a moving point cannot transmit the whole of its motion to a second point unless the latter is free to move in the same direction as the first. If it can only move at right angles to the direction of motion of the first, then it can receive no motion at all from the moving point, and if it can only move at an acute angle, then its motion depends upon that angle. Evidently that is the case here, for the directions in which points on the driving and driven wheels move only coincide when the points are in contact at C. At other

times the entire motion of a point on the driver cannot be imparted to a point on the driven wheel, because they are not moving in the same direction.

Now, consider wheel A to have teeth formed by cutting radial grooves, so that the radius AH represents one side of a groove, and the wheel O to have teeth formed on it, which act to drive A by engaging the side of the radial groove. When the radius coincides with the line of centers AO the point where the tooth on O touches it must be at C, as we have seen, but we do not yet know where the point of contact should be when the radius is in some other position, such as AH. We know that the direction in which the point of contact is momentarily moving must be at right angles to the radius, since a point revolving around a fixed center is moving at any given instant in the direction of the tangent to its path at that point, so in order to assume some definite position for the point of contact let us assume that it lies at B,

where the perpendicular from C' meets AH. Then the radius which determines the angular velocity of circle A is AB instead of AC, which being shorter, would cause A to rotate faster unless the effective radius of the driving point has been reduced proportionately. Draw OB and let fall a perpendicular from O upon BC produced, meeting it at D. Now, although the driving point at B is travelling around the center O at an increased speed owing to its greater radius OB, it cannot transmit all its velocity to the driven point at B for the reason stated above, that they are not moving in the same direction. As far as motion in the same direction—that is, along line CB—is concerned, it can be seen that the effective radius of the driving point B is not OB but OD, which is perpendicular to the direction of motion CB, so that the effective radii of the points in contact are now AB and OD instead of AC and OC. The proportion between each of these pairs is, however, the same, to prove which a little ele-

mentary geometry must be considered. The triangles ACB and OCD are similar, each of their angles being respectively equal, and therefore their corresponding sides are proportional—that is, $AC : OC = AB : OD$. We saw that the angular velocities of the smooth wheels were proportional, inversely, to the radii AC and OC; therefore the angular velocities are unchanged when the wheels are toothed in the way we have assumed, because the new radii of the points in contact have the same ratio as AC and OC. Now, clearly this ratio would be changed if the perpendicular to AH at B did not pass through C, because then the triangles would not have a pair of sides equal in length to AC and OC, so we have discovered one fact—namely, when a tooth of one wheel engages the side of a radial groove on the other wheel, the point of contact must always be in the perpendicular let fall upon the radial face from the point of contact of the two smooth pulleys, the motion of which

is to be reproduced by the toothed wheels.

When two curves touch each other in one point only without crossing each other they have a common tangent at that point, and the perpendicular to the tangent at that point is the common normal. Evidently if one of the curves is a straight line it coincides with the tangent, and the perpendicular to it at the point of contact is the common normal. Therefore *BC* is the common normal to the tooth faces, and the condition for uniform motion is that the common normal at the point of contact of a driving and driven tooth must pass through the line of centers at the point of contact of the pitch circles, for the pitch circles of a pair of gears are those imaginary circles which are tangent to each other when the gears are in mesh, the radii of which are inversely proportional to the angular velocities of the gears.

In Fig. 2 successive positions, AH^1, AH^2, AH^3, of the radius AH and of the

perpendicular to it through *C* have been drawn, and we find that the path de-

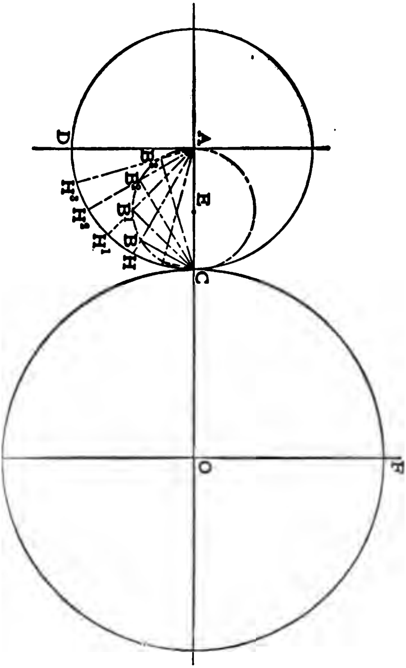


Fig. 2.—Principles of Gear-teeth Curves.

scribed by the point of contact on a stationary plane—that is, one not moving

with the rotating wheel A—is a circle whose diameter equals the radius of the circle A. Geometrically this is proved by the fact that the locus of all points at the apices of right-angled triangles having a common hypothenuse is a circle of which the common hypothenuse is a diameter.

We want to find, therefore, the form of tooth on O which will drive A by contact with the radius AH at the successive points indicated by B, B^1, B^2 and B^3. Now, it is a remarkable fact that if the small circle ABC is rolled around on the inside of the circumference of circle A a point on its periphery, such as B, will travel along a radius of circle A, thus tracing the very form of tooth-face which we have chosen for that wheel. This can be easily demonstrated, and first it must be noticed that the relative motion of the two circles is the same whether the larger is stationary and the small one rolls on its periphery, or whether they both rotate on fixed centers and one drives the other by friction at the point of contact, for in

each case the periphery of the small circle is "laid down," or "measured off," on the periphery of the large circle, and every successive point of one circumference comes into contact with every successive point of the other. This is the case represented in Fig. 2. The circle *A* in rotating has carried the radius *AH* from a position coinciding with *AC* through the various positions AH^1, etc., to a position coinciding with *AD*, and at the same time the small circle in rotating has carried the point *B* from *C* through B^1, etc., to *A*, and it will be noticed that every position of *B* lies in the radius *AH* in its corresponding position.

To demonstrate this fact, let *AH* in Fig. 3 represent the position of the radius *AC* of circle *A* after it has rotated through an angle *a*, and assume that *ED* represents the position of the radius *EC* of the circle *E*, which has been rotated through an angle *b* by frictional contact of the circumferences at *C*.

We know that the arc CH = arc CD, since the points D and H have moved with equal velocity from C, where they coincided to their present positions. We

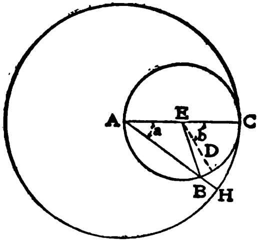


Fig. 3.—Principles of Gear-teeth Curves.

also know that the angular velocities of the two circles are inversely as their radii, and that, therefore, angle $b = 2$ angle a.

Now, the angle at the center of a circle which subtends a given arc is equal to twice the angle at the periphery which subtends the same arc. Therefore, since b is an angle at the center and a an angle at the periphery of the circle E and b is twice as large as a, the two angles must subtend the same arc on that circle, and

D must coincide with *B*, the point where *AH* intersects the periphery of *E*. Since the angle *a* represents any angle through which *AH* may be turned, and the angle *b* must always equal 2*a*, it follows that any point *B* on the circle *E* traces as it rolls inside it a radius of the circle *A*.

A beautifully simple way of drawing the proper form of tooth on the other pitch circle *O* is now available. Take the same circle *E*, which when rolled inside pitch circle *A* traces the tooth-face *AH*, and roll it on the outside of pitch circle *O*, as in Fig. 4, a point on its periphery such as *B* will trace a curve *CBD* which is an epicycloid. The motion being the same relatively, whether *E* is rolled around *O* or both are rotated on fixed centers at equal peripherial speed, it may be considered as having simultaneously (see Fig. 2 again) a rolling motion inside circle *A* and outside circle *O*. The same point on its periphery, then, that traces the radius *AH* of *A*, traces the epicycloid *CBD* on *O*, consequently that

epicycloid is the form of tooth suitable to drive the radially grooved wheel, because at every instant the engaging

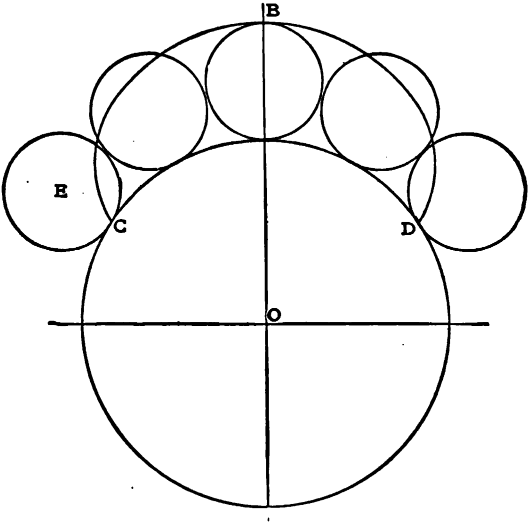


FIG. 4.—Principles of Gear-teeth Curves.

surfaces have a common point of contact (since they are traced simultaneously by the same point), and the common nor-

mal at that point of contact passes through the point of contact of the pitch circles.

Suppose a gear of greater diameter than *A* is wanted which will mesh equally well with *O*, the form of its teeth may be determined by the same circle *E*. A point on the periphery of this latter will, however, no longer trace a radius of the circle in which it is rolled, but a hypocycloidal curve such as *BFG*, in Fig. 5, which will be the proper form of tooth to mesh with the epicycloidal tooth *DFH*, traced by the same point, on wheel *O*. That the common normal to the two curves at the point of contact, such as *F*, will pass through *C*, the point of contact of the pitch circles, is evident when one remembers that any cycloidal curve may be considered as made up of a series of infinitesimal arcs of circles, the momentary centers of which are the successive points of contact of the rolling circle and the line on which it rolls. Therefore both curves at *F* coincide with an arc whose

center is C, and the radius CF is the common normal. Of course, ordinarily gears have both epicycloidal faces and hypocycloidal flanks, so that they may be

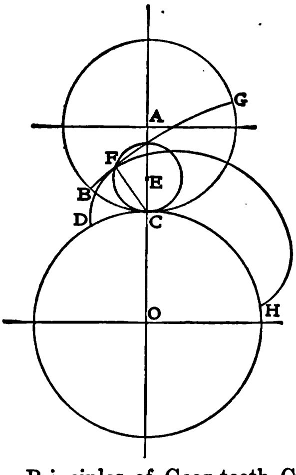


Fig. 5.—Principles of Gear-teeth Curves.

used indiscriminately as the driver or as the driven wheel.

The other form of tooth in common use is the involute, which curve is traced by a point on a string, which is kept taut as

it is unwound from the periphery of a circle. This is the same thing as though the tracing point were on a tangent line which rocked around the periphery. Thus in Fig. 6, considering the circle *A* only for the moment, the point L, which traces the involute *LF* on the circle *A* may equally well be considered as on a string *MQL*, which is wrapped around the circumference, or as on a tangent line *SQL*, which is rocked around the circumference. In either case the tangent point *Q* is the momentary center of the involute curve at *L*, which, like cycloidal curves, may be considered as made up of infinitely short circular arcs, the centers of which are the successive points of contact of the rolling line with the circle. Indeed the involute is a limiting case of epicycloid, the diameter of the rolling circle having become infinite and its periphery consequently a straight line.

It is also true that the relative motion of the tangent line and the circle is the same in the three following cases:

First—If the circle is held stationary and the line is rocked around it.

Second—If the line is held stationary and the circle is rolled upon it; and

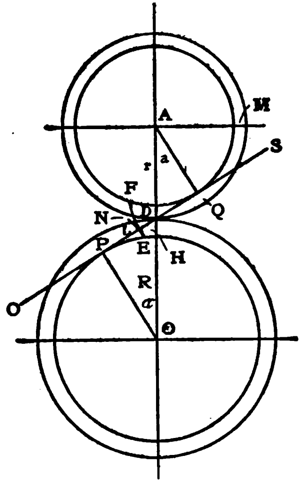


FIG. 6.—Principles of Gear-teeth Curves.

Third—If the circle is rotated about a fixed center and the line is moved in the direction of its length at the same linear velocity as the periphery of the circle.

If this does not appear obvious at first glance, consider a rack and pinion in mesh. Clearly the action between them is unchanged whether the pinion is stationary and the rack swung around it or the rack stationary and the pinion rolled upon it, or the pinion rotated and the rack moved longitudinally.

Owing to the fact that the same relative movements serve for the describing of a cycloid, it is not perfectly easy to see that the second and third cases serve for the describing of an involute just as well as the first. This is because the circle is moving and the involute to it cannot be drawn upon the stationary plane of the paper. It must be remembered that the tracing point of the cycloid is on the circle and that the tracing point for the involute is on the line; also that a single point can trace two or more curves simultaneously upon two or more planes which coincide in space, but one or more of which has a movement of rotation or

translation, or of both rotation and translation together.

That the involute will be traced in space whether the tangent line is rocked around the circle or the circle rolled along the line can be easily proved by analytical geometry, and the following demonstration is offered as being of possible interest to some (see Fig. 7): In the left hand figure LU is a portion of the involute to the circle OLQ of radius r, traced by the point L when the tangent line LW has been rocked to the position UQ, in which it makes an angle θ with its first position. The co-ordinates of the point U with references to the axes X and Y, with origin at the center of the circle, are as follows:

$$x = TU = TV - UV,$$

$TV = ON = r \cos QON = r \sin \theta$, because $2ON$ is the complementary angle to OQN, which is equal to LSU (θ), their sides being at right angles to each other; and $UV = UQ \cos \theta$, because $QUV = LSU$ (θ),

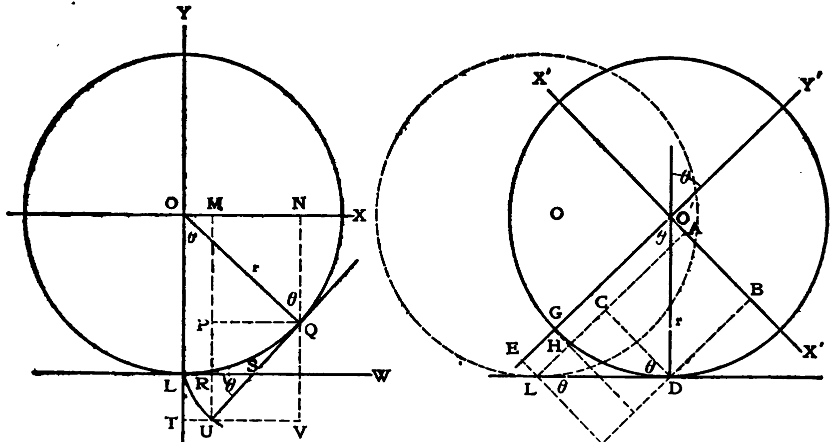


Fig. 7.—Principles of Gear-teeth Curves.

being opposite internal angles; $UQ=$ the arc LQ, because each consecutive point of the line and arc have been in contact as the line rolled from its original position LW to its present position UQ, therefore $UQ=r\,\theta$ since that is the measure of the arc LQ, and the angle $LOQ=LSU$ (θ), their sides being at right angles, hence

$$UV=r\,\theta\cos\theta,$$

and

$$x=r\sin\theta-r\,\theta\cos\theta.$$

Considering y as positive, as will also be done in the second case,

$$y=MP+PU$$

$$MP=NQ=r\cos\theta$$

and

$$PU=QV=UQ\sin\theta=r\,\theta\sin\theta,$$

hence

$$y=r\cos\theta+r\,\theta\sin\theta$$

In the right hand figure the line remains stationary and the circle rolls. We want to prove that a stationary point on the line will trace the involute to the rolling circle. The plane on which the involute is traced moves with the circle, consequently the equation of the involute will be given by the co-ordinates of the tracing point with reference to axes which move with the circle.

L is the tracing point and $X'Y'$ the position of the axes X and Y after the circle has been turned through an angle θ which, since there is no slip between the circle and line, has carried it from the dotted line position to the full line position, the distance between the two positions being obviously equal to $r\theta$. The co-ordinates of the point L with reference to the axes $X'Y'$ are then

$$x = EL = EF - LF,$$

$$EF = O'B = r \sin \theta,$$

and

$$LF = LD \cos \theta = r\,\theta \cos \theta,$$

hence

$$x = r \sin\theta - r\theta \cos\theta.$$

$$y = AC + CL,$$

$$AC = BD = r \cos\theta,$$

and

$$CL = DF = LD \sin\theta = r \sin\theta,$$

hence

$$y = r \cos\theta + r\theta \sin\theta,$$

which proves the proposition, since the co-ordinates of the stationary tracing point with reference to the axes of the rolling circle are the same as the equation of the involute with reference to the axes of the stationary circle.

The involute forms both the faces and flanks of the teeth, which enables a pair of gears with this form of teeth to possess the valuable property of meshing equally well when the distance between their centers is increased over the sum of the pitch circle radii. In other words, they have no definite pitch circle diameters aside from

those determined by the ratio of their angular velocities, which of course depends upon the number of teeth. The involutes are not drawn to the pitch circles, that is, to the imaginary smooth surfaces which would transmit the same angular velocities, because then depressions or spaces inside the pitch surfaces would have to be provided, and all the advantage incident to the use of involute teeth would be lost, as the teeth would then be double-curved, like epicycloidal teeth, and the center distance could not vary, but they are drawn to concentric circles of less diameter than the pitch circles. The diameter of one of these circles may be chosen arbitrarily, which determines the diameter for the other gear of the pair, or set.

Turning back now to Fig. 6, the circles which touch each other at the point D represent the imaginary smooth surfaces whose relative angular velocity the gears are to maintain; in other words, they are the pitch circles.

AQ is a line drawn at an arbitrarily chosen acute angle *a* to the line of centers *AO*, and *DQ* is the perpendicular from *D* to *AQ*. *OP* is the perpendicular from the center *O* of the other pitch circle to *QD* produced. The circles described with radii *AQ* and *OP* are the base circles of the involutes, and *PQ* is their common tangent.

Now if both circles are rotated at equal peripheral speed in opposite directions the tangent *PQ* will move at the same speed in the direction of its length, and a point such as *L* will simultaneously trace the involutes *FLH* and *NLE* to circles *AQ* and *OP*, respectively. The same thing takes place when *PL* and *LQ* are considered as portions of strings wound around their respective circles, and with their ends meeting at *L*. The involute *NLE* would be traced by winding *PL* back on circle *OP* and involute *HLF* by winding *QL* back on *AQ*.

These involutes will fulfill the requirements for mating tooth surfaces, because, first, they have a common normal *LQ* at

their point of contact, and, second, the common normal intersects the line of centers at the point of contact of the pitch circles.

As the angle *a* may be chosen arbitrarily, an infinite number of involutes can theoretically be drawn that will fulfill the conditions, but experience has determined the most efficient size to give to that angle. It is, however, the complementary angle *ADQ* which the tangent to the base circles makes with the line of centers which is determined upon. This angle is called the angle of pressure, since it is along the common normal that the force exerted to rotate the driven wheel acts, and for an interchangeable set of gears is taken as 75 or 75½ degrees. A more obtuse angle would tend to reduce the height of the teeth too much, and a more acute angle would cause the thrust component of the force to become inconveniently great; *i.e.*, the force acting to separate the gears along the line of centers, which component also causes

sliding movement between the tooth surfaces. This angle is frequently stated in terms of its complement which is commonly called the angle of obliquity. William Sellers & Company use an angle of obliquity of 20 degrees, and Wilfred Lewis considers that this could be advantageously increased to 22½ degrees. The advantage of the larger angles of obliquity (smaller angles of pressure) is that they overcome interference of the teeth of low numbered pinions when working with high numbered gears.

Formerly gears with involute teeth were looked upon with disfavor, because it was thought that the pressure upon the bearings was much greater than with epicycloidal gears, but by experiment and actual experience this has been proved to be a fallacy, the pressure being hardly perceptibly greater when the angle of pressure is not more acute than 75 degrees, and involute gears are largely superseding the epicycloidal, or, more correctly, cycloidal gearing.

CHAPTER II.

THE FORMATION OF GEAR TEETH.

HAVING now briefly sketched the manner in which the forms for the teeth are determined, the next question is how to put the theory into practice; in other words, to investigate the actual machines which have been designed and built to cut epicycloidal or involute gears.

The direct application of geometry to the problem of the action of gear teeth was shown in 1675 by M. de Roemer and in 1695 by M. de la Hire, followed in the first half of the eighteenth century by M. de Lalande and M. Camus, all four French mathematicians and scientists, but it is only within the last fifty years that gear-cutting by machinery has been at all common and within the last twenty years or thereabouts that generating machines proper have come into use. The bicycle

fever was responsible for the invention during the last decade of the nineteenth century of a considerable number of generating machines designed especially for the manufacture in large numbers of very accurate bevel gears.

The necessity of shaping the teeth of wheels according to the principles of geometry seems to have been first appreciated by watch and clockmakers. A number of works on horology published 50 to 100 or more years ago contain chapters devoted to the theory of gear teeth, and it appears that the first actual embodiment of the generating principle in a machine was for the purpose of finishing the teeth of watch wheels, as will be seen later. Since gears have been made by cutting the problem of correctly shaping the teeth has been met in three ways:

First—By causing a formed cutter to so move relatively to the blank as to cut a groove of the same cross-sectional shape as the cutter, the sides of which groove

are the adjacent sides of two adjacent teeth. This method is the one in common use for cutting spur-gears.

Second—By causing the cutter to reproduce in the blank the shape of a previously formed templet, or master-tooth. This was the only method of cutting theoretically correct bevel gear teeth until generating machines were invented.

Third—By causing the tool to generate the tooth curves by its motion relatively to the blank. This method is applicable to the production of spur, spiral and bevel gearing, but is more particularly useful in the cutting of bevel gears.

The first two methods involve the preliminary formation of a templet, in one case to be used in the gear-cutting operation and in the other to give the correct contour to the formed cutter, the templet being used in a pantographic milling machine, so that the accuracy of the gears made by both these methods depends upon the correctness of a templet. These may be, and formerly were

always, shaped by hand, but now it is usual to employ generating machines to form the templets, to certain of which reference will be made again later.

While very accurate teeth can be made by these methods, they are not as simple nor as beautiful in principle as the third method, which does not require the preliminary formation of any templet, yet which produces teeth so accurate that any greater refinement would be wasted, because the rest of the machine in which such a gear was used would not be accurate enough to warrant it.

As the title indicates, this review treats only of machines working on the third or generating principle, which machines, however, will not be described in detail, since we are here only concerned with motions and not with forces. It is the intention to show, after some general explanation of how the theory outlined above may be put into practice, only the mode of operation of the machines; in other words, how the relative motions

of tool and blank, which are essential to produce the true tooth-curves, are obtained.

Descriptions and illustrations of several of the more recent machines have appeared separately in the *American Machinist,* and when such is the case reference to the date of the issue containing such description will be made.

Mr. George B. Grant, in an article in the *American Machinist* for June 7, 1894, defined the verb "to generate" as "to form by virtue of the principle of operation of the machine rather than to form a copy of a previously formed templet or cutter." The term must be restricted in its use to the operation of mechanism which produces surfaces which are neither planes nor surfaces of revolution; otherwise the above definition would be broad enough to cover ordinary turning and planing. Applied to gear generating machines, it of course means giving the tool such a motion relatively to the blank as will cause it to correctly shape teeth

thereon, the shape of the cutter not being that of the spaces formed nor any templet being employed.

Referring to the methods described above of laying out or tracing cycloidal and involute curves, it will be evident that tooth surfaces of those forms can be produced by giving the point of a tool the same relative motions to the pitch circles as the tracing points have, and in addition a cutting movement perpendicular to the plane of those circles. For example, the point of a tool reciprocated along an element of a cylinder that was being rolled at a relatively slow rate on the surface of another cylinder would generate an epicycloidal surface, such as *ABCD* in Fig. 8. If the small cylinder were rolled inside the large one the element would evidently generate a hypocycloidal surface. Also, if a plane were rocked on the pitch surface, a cutting point reciprocated along a line in the plane parallel to the axis of the pitch surface would generate an involute sur-

face, as *EFGH* in Fig. 9. The pitch and rolling cylinders are replaced by cones

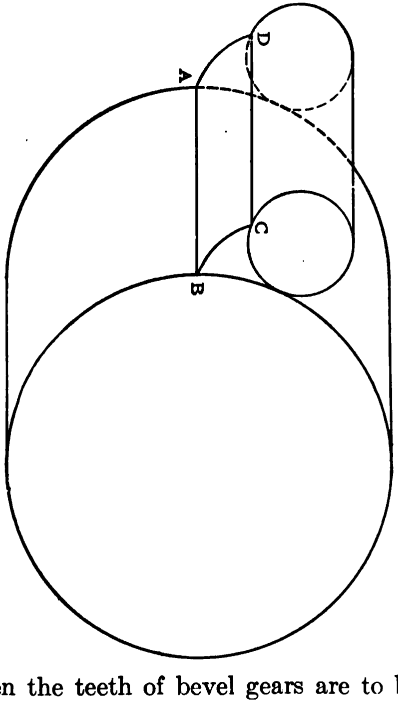


Fig. 8.

when the teeth of bevel gears are to be cut.

To this way of shaping the teeth the writer has given the name of "the de-

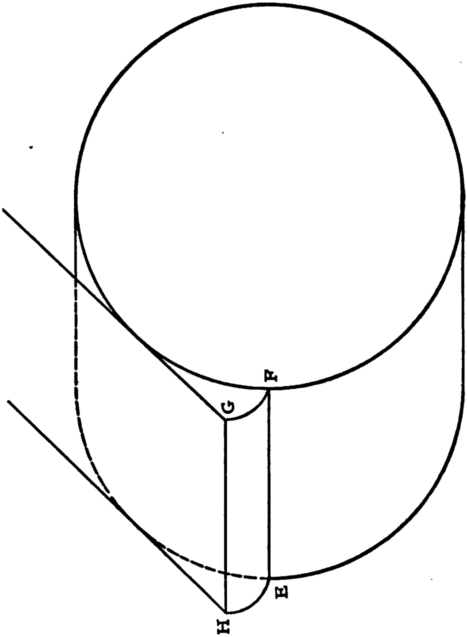


FIG. 9.

scribing method," since the curved surfaces are generated by the cutter point

working in three dimensions exactly as the pencil point describes the curves in a plane. There is another method of generating gear teeth which I call "the intermeshing method," and its theory of action can be readily understood, even without any knowledge of the geometry of the tooth curves.

Suppose a wheel of some hard material having a boss or projection at one point on its periphery is rolled against the periphery of a second wheel of plastic material or heated to a plastic condition. A depression will be formed in the periphery of the second wheel, the shape of which will evidently depend on that of the projection; in other words, it will be "conjugate" to it. In Fig. 10 is represented the formation of conjugate depressions by a series of projections. This must be true whatever the form of the projection, and if we gave it that of a gear tooth, suitable to transmit uniform motion, such as one of the two forms in common use, epicycloidal or involute, the depression

in the second wheel will be correspondingly or rather "conjugately" shaped, so that together they will form a perfect pair of mating teeth. In the same way a complete gear could be formed by using a complete gear as the generator,

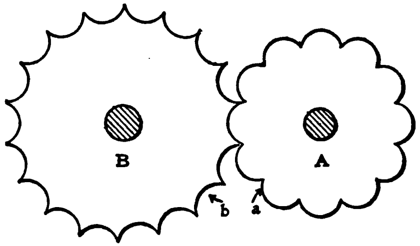


FIG. 10 —Principle of the Molding Process.

provided the periphery of the blank is divisible by the circular pitch, or distance between similar points on two adjacent teeth of the generating gear. In order that the gear so formed may be used as a driver it must of course have projections as well as depressions, that is, considering its original diameter as that of the pitch circle, so that the gener-

ating gear should have depressions of the proper form between its teeth, into which the metal of the blank, supposing it to be sufficiently plastic, would be forced by the pressure of the adjacent teeth. Or if the gears are rotated by outside means, as if their pitch circles were in contact, then the blank may have an outside diameter equal to that of the "addenda" circle of the finished gear, or over all dimensions to the top of the teeth.

This process is known as the "molding process," and is disclosed in a United States patent of 1872 granted to John Comly, of New York. Unfortunately it is not practicable; if it were gear cutting machines would have to be put out of commission, since it would easily be the simplest and quickest way to make gears. The principle underlying it, however, the first announcement of which antedates the Comly patent by over twenty years, can be and is employed in the generation of gear teeth. This principle may be stated as the form-

ing of conjugate teeth on a gear-blank by running it in mesh with a gear; hence the name "intermeshing." It is only necessary, as with the describing method, to give the tool an additional cutting movement at right angles to a normal section of the tooth, the relatively slow or intermittent intermeshing movement being maintained, in order to cut out teeth in the form they would have if made by the molding process unmodified. The cutter need not of course be a complete gear; it may represent only one tooth of the generating gear, or even merely one side of a tooth, but in the two latter cases indexing mechanism must be provided to bring successive portions of the periphery of the blank into relative position to the cutter. The additional cutting movement of the tool may be rotary instead of reciprocatory, or it may consist of both, as where a rotating cutter is fed across the face of the blank.

The principle remaining the same whatever the size of the generating gear,

it is evident that the cutter or cutters may represent the side or sides of a tooth or teeth of a rack, and there are certain advantages gained thereby which have caused the rack-tooth form of generating cutter to be frequently employed by inventors of machines using the intermeshing method. The principle advantage is that by its use a complete set of interchangeable gears can be made that will mesh perfectly with each other. This theory, applied to the laying out of templets, was first announced and discussed in a treatise published in 1850, the author being an Englishman of the name of Edward Sang, and is known as "the Sang theory."

The reason that the rack-tooth cutter is essential to the generation of a set of interchangeable gears is that a generator cutter in the form of a gear cuts out only just enough clearance at the roots of the teeth to enable it to pass, so that that gear when finished will not run properly with any others of the same set which

are larger in diameter than the generator. The fillets could of course be reduced by filing, or, as is done by one inventor, by increasing the height of the teeth of the generator from the pitch circle outwardly.

Another advantage obtained by the use of the rack tooth lies in the fact that in the involute system it has plane sides, which are easy to form accurately, while if an involute or epicycloidal *gear* tooth be used as the generator a generating machine must be used to form it accurately.

The Sang theory can easily be tested experimentally by taking a board having a straight edge to represent the pitch surface of the rack, and attaching to it so as to protrude over the straight edge a pasteboard or thin sheet metal templet of the form of an involute rack tooth, leaving a space under the templet a little way in from the straight edge, so as to provide room for the disk representing the addendum circle of the mating gear to pass.

The involute rack tooth is a truncated wedge, with sides at about 75 degrees to the straight line forming the end of the tooth. The pitch circle of the mating gear is represented by a disk of wood or other suitable material of the same thickness as the board, to which is pasted a segment of thick paper to represent the outside diameter of the gear teeth. Then roll the disk without slip on the straight edge and repeatedly scribe around the templet on the protruding segment. A number of successive positions of the rack tooth relatively to a gear blank of the diameter chosen will thus be determined, and by drawing the curves or "envelopes" enclosing all these positions the proper form for the conjugate teeth will be obtained. A neat device of this kind, where two gears were, however, represented instead of a gear and a rack, was illustrated in the *American Machinist* at page 176, vol. 23. The accompanying sketches (see Fig. 11), taken from a patent for an involute-curve

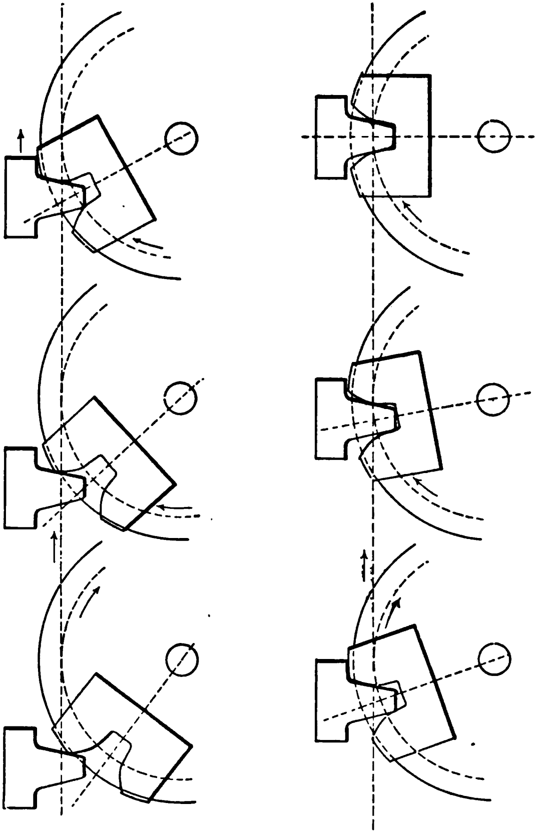


FIG. 11.—Formation of a Gear Space from a Rack Tooth.

shaping-machine, described hereinafter, illustrate, reading from left to right, the successive relative positions of the rack and gear teeth.

When an interchangeable set of bevel gears are to be formed by the intermeshing method and Sang theory, the tool should represent a tooth of a crown gear, since that is the limiting form for bevel gears, as the rack is for spur gears. Although the sides of a crown gear tooth of the involute system are not strictly plane surfaces, as are those of the involute rack tooth, it may be assumed that they are and a cutter be used to generate bevel gear-teeth, which has the shape of the rack tooth instead of that of a common gear. This assumption results in the formation of so-called "octoid" teeth on the bevel gear, which are very similar to true involutes and work just as well. As stated by Mr. Grant in his treatise, the line of action of a pair of octoid teeth on the normal spherical surface is not a great circle of the sphere, as in the case of invo-

lute teeth, but makes a figure resembling a double loop or figure 8, hence the name.

Owing to the difficulty of producing accurate bevel gears by ordinary methods the generating method is particularly suited to this kind of gear. Most of the more recent generating machines are designed especially for bevel gear work, and most of them form "octoid" teeth; in fact, along those lines appears to lie the trend of development of the art.

Having thus outlined the principles of operation of machines using the describing method and those of machines using the intermeshing method, we will consider a class of machines the mode of operation of which may be considered as involving either or both methods, viz., those for generating gears to drive a "pin wheel" or "lantern gear," that is, a gear having teeth of the form of cylindrical pins or staves arranged parallel to its axis, the axis of the pins all lying in the imaginary pitch surface.

Suppose, first, that these pins were of infinitely small diameter, that is, mere points on the pitch circle, considering an end elevation. The pitch diameter of the gear which is to drive the pin gear being chosen, the proper form of teeth which it should have will be determined by the curve traced by one of the points when the pitch circle of the pin gear is rolled on the pitch circle of the gear to be formed, which will be of course an epicycloid. So far the describing method has been employed. Since the pins must have a sensible diameter, if the points on the pitch circle of the pin gear are replaced by small circles of the diameter of the pins and the same operation repeated the tooth outline formed will be a parallel epicycloid to the first at a normal distance from it, equal to the radius of the pin. Obviously also a clearance curve will be formed inside the pitch circle of a depth equal to the radius of the pin. Supposing the pin circle to represent a rotating milling cutter,

it is evident that the proper form of tooth ean be milled out, which would be an instance of the employment of the intermeshing method.

CHAPTER III.

THE FIRST GENERATING MACHINES.

THE second earliest generating machine of which any record has been found was of the type last described, and is illustrated in Fig. 12, which was taken from the *Mechanics' Magazine*, of London, for January, 1849. The inventor was F. Bashworth, Esq., of St. John's College, Cambridge. There is no evidence to show that he actually built a machine or a working model. The operation is precisely that described above; E is the milling cutter, F the gear teeth being shaped, A and B friction wheels of diameters equal to the pitch circle diameters of the pin gear and gear being shaped respectively. At C are represented bands to prevent slip between the friction wheels. The small figure on

the upper right-hand corner illustrates the describing of epicycloids by a point on the pitch circle of the pin gear, and that on the lower left-hand side the de-

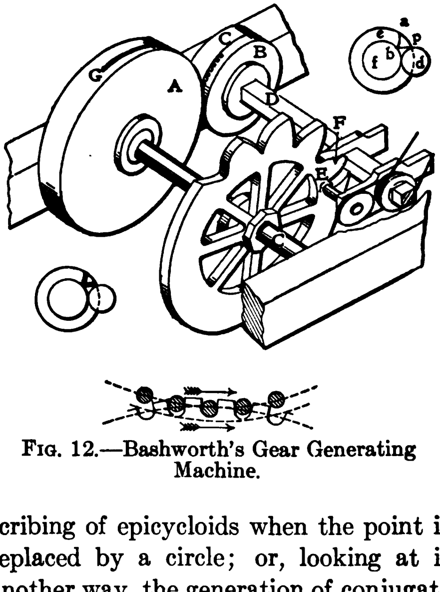


FIG. 12.—Bashworth's Gear Generating Machine.

scribing of epicycloids when the point is replaced by a circle; or, looking at it another way, the generation of conjugate teeth by the intermeshing method. The

lower central view shows the actual form of the teeth as used in practice, only very short parts of the epicycloids being employed.

The only description of a generating machine which antedates that of Mr. Bashworth's was found in the appendix to the 1842 edition of Hawkins' translation of Camus' treatise on the teeth of wheels. The generating engine described was, as will be seen, of American invention, and was designed to shape the teeth of watch wheels.

Since no illustration of it accompanied the description, the writer has attempted to show in Fig. 13 what the arrangement of parts must have been.

The quotation is as follows:

"Mr. Saxton, of Philadelphia, now in London, who is justly celebrated for his excessively acute feeling of the nature and value of accuracy in mechanism, and who is reported not to be excelled by man in Europe or America for exquisite nicety of workmanship, made

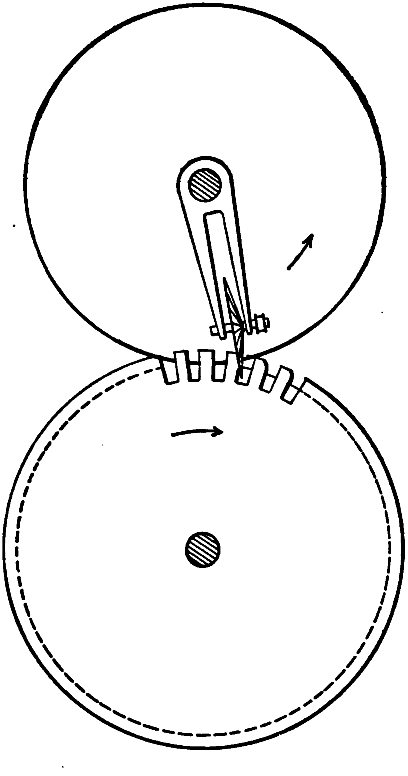


Fig. 13.—Saxton's Gear Generating Machine—1840.

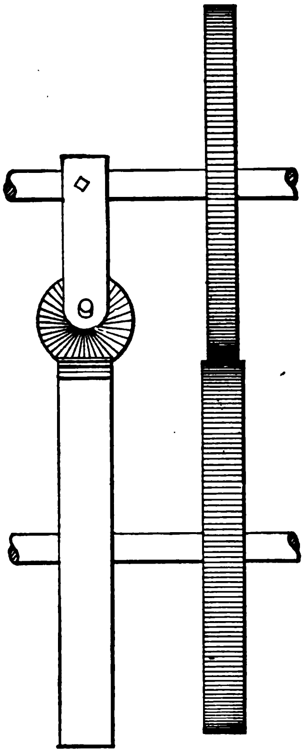


Fig. 13.—Saxton's Gear Generating Machine—1840.

in Philadelphia an instrument for cutting the teeth of watch wheels, truly epicycloidal; or, rather, for curving them after they were cut down in the ordinary manner with radial faces. The following is his verbal description of this instrument:

"The wheel to be rounded being put on a vertical arbor, another arbor stands parallel to the first, carrying on a third but horizontal arbor a steel wheel file cut on the plane side, which plane side lies in a vertical plane passing through the axis of its vertical arbor. On the arbor of the wheel to be rounded is a circular plate equal in diameter to the primitive circle of that wheel. The edge of the plate is milled into teeth as fine as possible. This plate forms the base of the epicycloid. On the other vertical arbor is a similar plate, but equal in diameter to the radius of the primitive circle of the wheel to be engaged with that about to be rounded. This plate is the generating circle.

"In working this instrument the flat-

sided cutter is brought in contact with the side of the tooth to be rounded, the axis of the two vertical arbors, the face of the cutter and the line of the tooth all lying in one vertical plane. The cutter being set into rapid motion by a band, the generating circle is rolled around the base, and thus one side of the tooth is rounded in a truly epicycloidal curve of the required dimensions.

"Upon this plan epicycloidal teeth of any magnitude might be cut with great expedition."

This machine presents the identical conditions of the first concrete case considered above, since the teeth formed have radial flanks and epicycloidal faces. The rolling or describing circle which determines the form of the tooth faces is equal in diameter to the radius of the pitch circle of the gear with which the gear being shaped is to mesh; hence those epicycloidal faces will mesh correctly with the radial flanks of that gear. Moreover, since the plane active face of the

cutter represents one of those flanks and is given the exact motion which it would have when driven by one of the tooth faces being formed, the machine is an instance of the use of the intermeshing method, where the flanks of the teeth of one gear form conjugate faces on the teeth of another.

Attention may be here called to the fact that the action of a hob in cutting a spur gear or worm wheel and that of a tool in the lathe cutting a thread involve the intermeshing generating principle. The following definition of the mode of operation of gear-generating machines of this type will be seen to be broad enough to include all forms of machines for cutting helical gearing.

Those machines for forming toothed gearing, in which a tool, shaped in normal section like a tooth of a gear with which the one being formed will mesh, has the same motion relatively to the blank as that tooth will have when in engagement with the teeth of the finished gear, either

with or without an additional cutting movement. This subject—*i. e.*, the analogy between thread-cutting and gear-generating, will be considered again a little further on in connection with the description of a machine for planing spiral gears which was patented in 1894, and which serves as a sort of "missing link" to clearly show the identity of principle.

It would make this chapter entirely too long to include descriptions of the various machines for cutting helical gearing, and it would be quite unnecessary, since all machinists and others who will read this article are doubtless familiar with their operation. Considered, as I think they must be, as coming under the general head of gear-tooth generating machines of the intermeshing type, they of course antedate the spur-gear generating machines described above by centuries, I suppose. The worm-wheel hobbing machine patented by Sir Joseph Whitworth in 1835, an illustration of which

accompanied a letter from Mr. Grant in the *American Machinist* for August 4, 1888, is, I believe, the first instance of the intermeshing method employed to cut gear teeth, as contrasted with screw threads. Another early example is the machine for cutting the Hindey worm, the exact date of which I do not know, but which is described and shown in Willis' "Principles of Mechanism," London, 1841, at page 164. In 1850 Professor Schiele took out a British patent for a machine for cutting the teeth of spur gears with a hob, the blank and the hob being geared together to rotate at the proper ratio. Francis H. Richards, Esq., informed the writer that the operativeness of such a machine for the purpose described was generally doubted, but that he tested the method on a Pratt & Whitney gear cutter and found that it worked perfectly, and that he has the gear that was made still in his possession.

The Swiss device for finishing watch wheels described by Mr. Haschka in a

letter published in the *American Machinist* at page 364, vol. 22, which he states was invented about forty years before by Pierre Ingold, of Biel, is an example of the intermeshing method, which is a very close approximation to the "molding process." The tool used was like a spur gear, having file surfaces on its teeth, and was run in mesh with the wheel to be finished. A different cutter would be necessary for each different pitch.

The extreme simplicity of the essential mechanism of machines using a hob-like cutter to cut gear teeth by the intermeshing method is well shown in a patent granted in 1871 to Henry Belfield, of Philadelphia, the drawing of which is reproduced in Fig. 4. *B* is the cutter and *D* the blank pivoted on a slide rest at *b*. The threads of the cutter act to feed the blank, while its cutting edges formed by the transverse grooves cut the conjugate teeth on the blank.

Thus far the machines described have all involved the intermeshing method,

but from this time on until 1884 all the machines of which record has been found work according to the describing method.

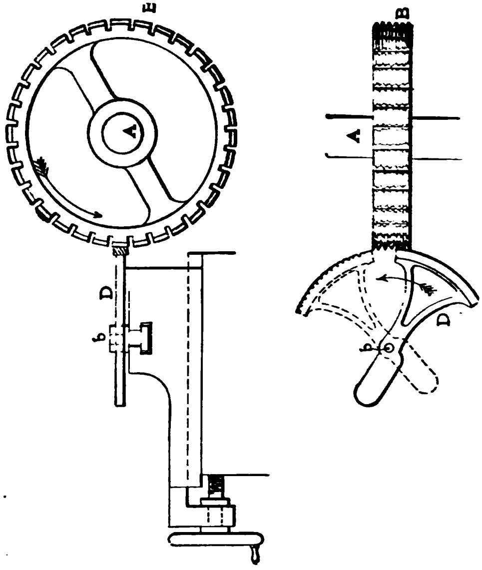


FIG. 14.—Belfield's "Intermeshing" Gear Current—1871.

It will, however, be noticed that in many instances, whether a machine works according to the one or the other method,

depends solely on the shape of the tool used; that is to say, whether the cutting edge is a point or as near to a point as practical considerations will permit, or whether the cutting edge represents the side of a tooth of a mating rack or gear. The relative motions are often identical, as where an involute is traced by a point on a line along which the base circle of the blank is rolled, for instance. The motion in that case is the same as that between a rack tooth and a gear, so that a machine using a cutting point, and therefore belonging to the describing type, would become an example of the intermeshing type if a tool shaped like a rack tooth were used. In epicycloidal machines the difference is more marked, since by the describing method the faces of the teeth only can be shaped by rolling the imaginary circle containing the point of the tool on the pitch circle of the blank, a different arrangement being necessary for cutting the hypocycloidal flanks. If, however, a cutter of the shape of an

epicycloidal tooth be substituted for the cutting point, then both faces and flanks can be finished at one setting. The imaginary rolling circle becomes in that case the pitch circle of a mating gear, but the relative motions of the two circles are the same whichever method is employed.

In 1859 an English patent for a machine using the describing method was taken out by two machinists of Leeds, John Lawson and Stephen Cotton. The general arrangement of this machine is sketched in Fig. 15, in which *B* is the vertically reciprocating tool-bar carrying the tool *T*, and supported adjustably on the crossrail *F*, which is rotatable in a horizontal plane, the ends being toothed to engage the worms *W* and guided in curved ways on the top of the uprights *U*. The worm shafts are geared together to rotate in unison and are connected by the train of change gears *L* with the shaft *K* for rotating the blank *G*. The two dotted circles in the plan view

represent the pitch circle of the blank and a rolling or describing circle respectively. To plane the tooth faces the cutting point is set at the point of contact of the two circles, and then the blank and crossrail are rotated by a crank applied to one of the worm shafts, the gearing being proportioned to cause the two circles to turn at angular velocities inversely proportional to their radii. For planing the flanks the operation is thus stated: "The cutting point of the tool must be extended beyond the surface of the rolling arc the distance of the pitch circle from the bottom of the tooth." It is evident that this would not serve to form hypocycloidal flanks, since the describing circle is still rolled around on the outside of the pitch circle. A point attached to a circle rolling on another circle, which point is located beyond the periphery, traces a form of rolled curve called a "curtate epitrochoid," which closely resembles the epicycloid traced by a point on the periph-

ery of the rolling circle in that part of it outside of the pitch circle, and forms a

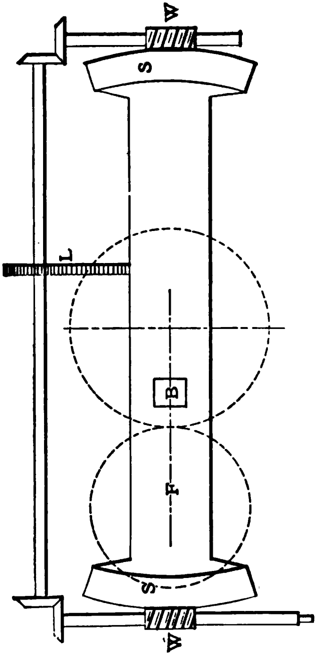


FIG. 15.—Lawson and Cotton's Gear Generator—1859.

concave, U-shaped loop inside the pitch circle.

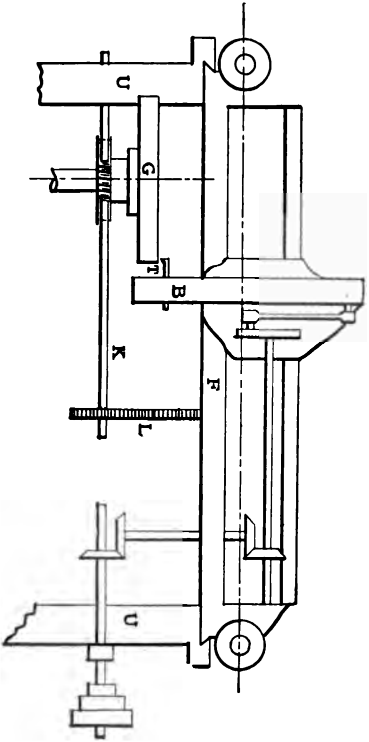


Fig. 15.—Lawson and Cotton's Gear Generator—1859.

That this method of forming the flanks should be employed appears strange, when, by adjustments toward the axis of the oscillating head or cross-rail *F* of the tool-slide and blank, it would be perfectly practicable to give to the cutting-point of the tool the motion of a point on a describing circle rolled inside the pitch circle of the blank, which, as we have seen, is the necessary motion to describe the flanks of teeth suitable to mesh with faces described by a point of the same circle when rolled outside the pitch circle of a gear of the same set. The same machine can be used to cut the teeth of bevel-gears by adjusting the guides for the tool-bar at an angle corresponding to the pitch-cone angle of the blank. The path of the tool then represents an element of a cone rolling on a conical pitch surface, instead of an element of a cylinder rolling on a cylindrical pitch surface.

The next two contributions to the art are in the form of treatises, published in

the transactions of two German societies, in which are discussed the principles involved in the production of theoretically correct gear-teeth, and which also describe and illustrate machines capable of carrying the principles into effect.

The first of these was found in the "Zeitschrift des Vereins Deutscher Ingenieure" for 1872. The author was E. Hagen-Torn, an engineer of St. Petersburg, and he shows how to generate involute teeth on both spur- and bevel-gears. He states that it was not his intention to furnish a complete plan of the machines with all details, but to show and state sufficient to enable any engineer to construct an operative machine. He proposed to use a shaper of the Whitworth type, and, for the generation of spur-gears, one in which the tool-holder had a lateral feed on the end of the ram. The two diagrammatic views of Fig. 16*a*, which is reproduced from the plate accompanying the article, illustrate the operation in cutting spur-gears. It will be

seen, from the shape of the tool, that the describing method is employed, for the cutting-edges are mere points.

The tool is reciprocated at right angles

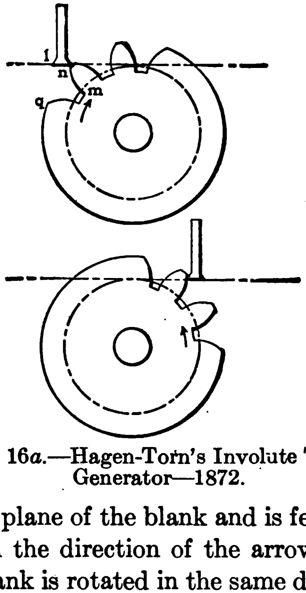


Fig. 16*a*.—Hagen-Torn's Involute Tooth Generator—1872.

to the plane of the blank and is fed laterally in the direction of the arrow, while the blank is rotated in the same direction with an equal linear velocity of pitch surface, the term pitch being used here for

the imaginary surface on which the involutes are formed, which, as pointed out

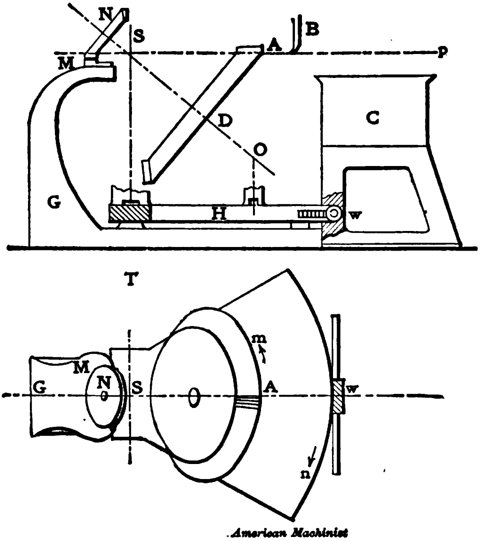


FIG. 16*b*.—Hagen-Torn's Involute Tooth Generator—1872.

before, is less in diameter than the pitch surface proper. The motion, therefore, of the cutting point relatively to the surface

is the same as that of a point of a tangent line which is rocked around the circle.

The tool may have two cutting points, *a* and *n*, and the width of the end *an* may be equal to the width *pm* of the tooth space at the pitch surface, so that after planing the face *nm* of a tooth from the top to the bottom, the feed being continued in the same direction, the corner *a* will plane the face *pq* of the next tooth from the pitch circle out to the top of the tooth.

After the space *nmpq* has been planed out in this way the machine is stopped, and can then be adjusted for the cutting of the next space in either one of two ways. If the spaces have been marked out on the outside periphery, then the tool and work are brought by hand into the position of the lower of the diagrammatic views, whereby the necessity of an idle return is obviated. Then the machine is reversed, so that the second space is cut during the return movement, as indicated by the arrow in said view.

According to the second method, the spaces do not have to be marked out, but the machine must be provided with an index-plate; so that the operation is, after one space has been cut, to return the tool and blank to their first position and then index the blank for the next space. For wheels of different diameter the ratio of movement must naturally be changed. For larger wheels, the author states, a vertical slotting machine is more convenient.

When bevel-gears are to be cut the problem is more complicated. The blank must be given a motion as if its conical pitch surface were rolling upon a plane, the apex point remaining stationary on the plane where the vertical axis around which the conical surface rolls meets the plane. Fig. 16*b* illustrates a machine for cutting bevel-gears, in which *C* represents an end view of the shaper ram guide and *B* the tool reciprocating on line *Sp*. The blank *D* must be given motions of rotation and transla-

tion as if its pitch surface were rolling on the horizontal plane containing line *Sp*. This is accomplished by mounting the arbor in fixed bearings at *S* and *O*, supported on a swinging carrier *H*, with the pitch-cone apex at the point *S*. The carrier is turned about the vertical axis *ST* by means of the segment and worm *w*. The motion of rotation is derived from that of translation by the engagement of the bevel-gear *N* on the blank arbor, with a stationary crown-gear segment *M* held on the standard *G*. For blanks of different pitch-cone angles the ratio between *N* and *M* must be changed. The planing commences at the outside periphery of the blank, working down towards the pitch circle. If the width at the point of the tool is equal to the width of the small end of a space at the pitch circle, then the second side of a space can be cut from the root up, but first the slide carrying the tool must be adjusted laterally the width of the tool end, and thus the second cutting corner

brought into line *Sp*. It will be noticed that, while the cutting corners of the tool describe the tooth curves, the machine would serve equally well for the intermeshing method if the shape of the tool had been that of a crown-gear or rack-tooth.

The other treatise is by Professor Hermann, of Aix, and appeared in the "Verhandlungen des Vereins zur Beförderung des Gewerbfleisses in Preussen" (Transactions of the Society for the Promotion of Industry in Prussia) for 1877. In it the author first discussed very fully the theory of gear-teeth, and then showed how machine tools in use could be modified to adapt them to generate cycloidal and involute teeth on spur, bevel, or hyperboloidal gears. The accompanying sketches, Figs. 17 to 21, are copied from the illustrations of the article, and will serve to show how Professor Hermann proposed to obtain the relative movements of the cutter and blank for each form of tooth. Thus Fig. 17 is a

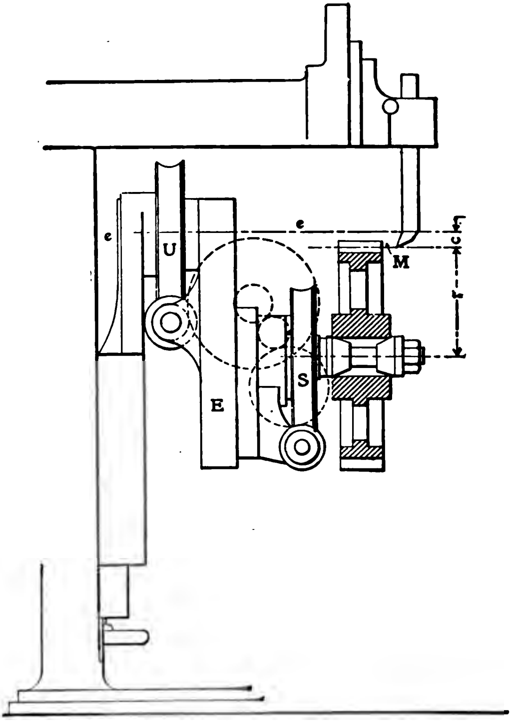


FIG. 17.—Hermann's Shaper for Generating Epicycloidal Teeth of Spur-gears—1877.

representation of a shaper provided with work supports capable of so presenting a spur-gear blank to the action of the tool that teeth having epicycloidal faces and hypocycloidal flanks may be planed out.

The blank is set with its pitch surface of radius $= r$, tangent to the path M of the tool, and is given a motion by means of the two worms and worm-wheels, S and U, as if its pitch surface were rolling on the surface of a describing cylinder of radius $= c$. This is accomplished by rotating the blank on its axis and at the same time swinging the carrier E around the axis ee, the angular velocities of the two rotations, which are fixed by the change gears shown in dotted lines, being, as in the case of the Lawson and Cotton machine, inversely proportional to the radii of the pitch and rolling circles, because the relative movement is unchanged by holding one circle stationary and rolling the other around it.

Adjustments are provided for different

sized blanks and describing circles, and to plane the flanks the blank and tool are adjusted vertically until the line M is at a distance $= c$ above the axis ee. The motion is then that of a circle rolling inside the pitch circle, the describing circle in this case being stationary and the pitch circle rolling on it.

The next view, Fig. 18, shows a shaper arranged to plane involute teeth on spur-gears. Here the blank is given a motion as if its base circle of radius $= r \sin 75°$ (if $r =$ the pitch circle radius and $75° =$ the angle of pressure chosen) were rolled on a horizontal plane in which lies the path M of the tool. This motion is compounded of a rotation of the blank on its axis and a straight-line lateral feed of the carrier F. In this case the worm-wheel S is therefore geared to the feed-screw G, which acts to slide the carrier. The length of the lateral feed must of course be equal to the arc on the base circle subtended by the angle through which the blank is simultaneously turned,

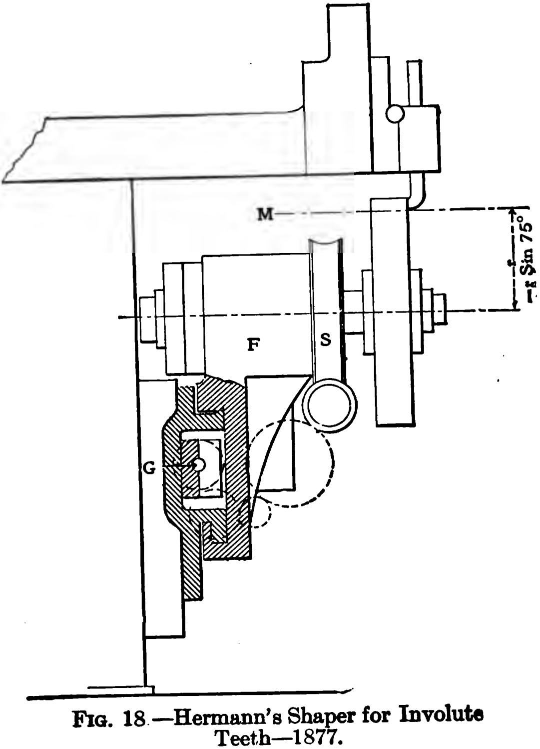


FIG. 18.—Hermann's Shaper for Involute Teeth—1877.

which depends, of course, upon the angle and the base circle radius. Thus if the angle = 30° and the radius = 6″, the lateral feed $= \frac{30}{360} \times 2\pi \times 6 = \frac{1}{12} \times 37.68 = 3.14''$.

For large gears the author proposed to use a vertical slotting machine, such as illustrated in Fig. 19. The change gears shown by dotted lines connect the worms of the worm-wheels S and U, as in the first shaper described above, the result being that epicycloidal teeth are planed out. For involute teeth the feed-screw G and worm-wheel S would be connected. The relative motions are precisely the same as in the shapers, only in a horizontal instead of a vertical plane; that is, for the epicycloidal surfaces the blank moves as though its pitch circle were rolling on the outside of a tracing circle whose axis is ee and radius the distance between axis ee and the path M of the tool, and for the hypocycloidal flanks the pitch circle surrounds and rolls with its

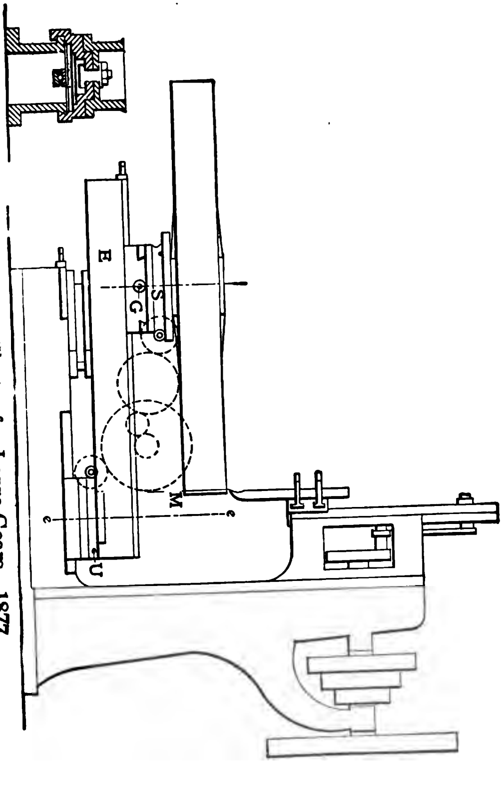
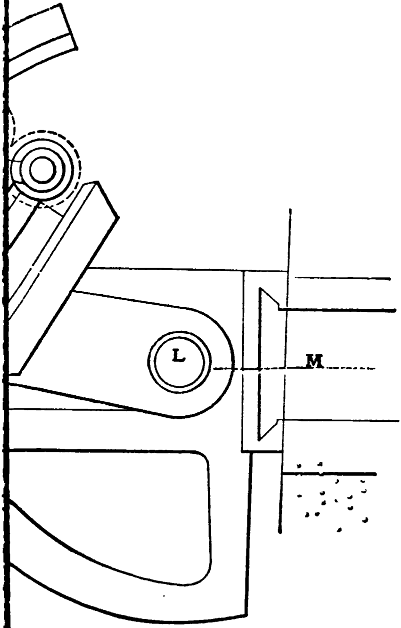


FIG. 19.—Hermann's Slotter for Large Gears—1877.

concave side on the periphery of the tracing circle.

Fig. 20 illustrates diagrammatically the arrangement of the work support in a shaping machine for planing epicycloidal teeth on bevel-gears. Here, of course, the pitch surface is a cone and must be given a motion as if it were rolling on a cone. *M* is the path of the cutting point, as before; *L* is the pivot of the blank carrier *H*, in the vertical axis of which the apex of the cone is set; *G* is a sleeve rotatably mounted on the carrier and carrying the worm-wheel *U* and the blank-supporting quadrant *Q* as well. The axis *C* of the sleeve *G* makes with *M* an angle equal to the half-angle at the apex of the tracing cone, and that between the axis of the blank and *M* is the half-angle of the pitch cone. The worm-wheels *U* and *S* are connected together as before, and serve to roll the blank around the imaginary cone surface. The adjustment for cutting the flanks is similar to that in the spur-gear shaper.



idal Shaper for Bevel-gears—1877.

Fig. 21 shows the arrangement when involute teeth are to be formed on bevel-gears, two of which appear in the drawing. Here, as in the Hagen-Torn machine, the blank is given a rolling motion on a horizontal plane containing the path of the tool by a simultaneous swinging around a vertical axis passing through the cone apex and rotation around its own axis, the worm-wheel *U* in this case being fixed on the shaper frame, or knee, and the worm being suspended from the carrier *E*, which swings around the bolt *C*. The remaining plate accompanying Professor Hermann's paper shows the arrangement for cutting involute teeth on hyperboloidal or "skew"-gears, which it would be useless to reproduce here unless the mathematical considerations were gone into, and it is not believed that it is worth while to take space for that, since such gears are not much used in practice.

Fig. 22 illustrates in outline a machine built by C. Dengg & Co., of Vienna.

patented in Austria in 1879, which generates epicycloidal bevel-gear teeth also

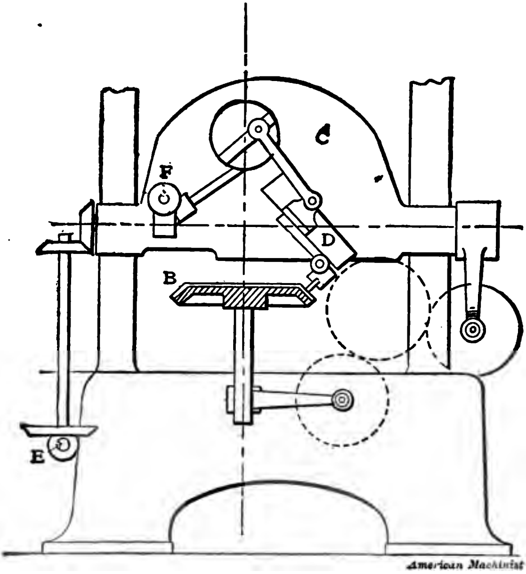


FIG. 22.—Dengg Epicycloidal Bevel-gear Generator—1879.

by the describing method. The blank *B* is supported on a vertical axis and the tool-carrier *C* is mounted on trunnions,

whose axis passes through the cone-apex point of the blank. On this carrier is an angularly adjustable guideway *D*, on which the tool-holder is reciprocated, power being transmitted from the driving-shaft *E* through spiral gears *F* to permit the oscillation of the carrier. The carrier and blank-mandrel are connected by change gearing so as to rotate in unison. The path of the tool represents an element of the tracing cone, the axis of which is that of the carrier, which cone touches along a line the pitch cone of the blank. It is not stated how the flanks of the teeth are to be formed, and the machine could obviously not be used to plane hypocycloidal flanks, since the tracing cone cannot be arranged inside the pitch cone of the blank.

Mention may here be made in passing of the epicycloidal milling engine invented by Mr. Ambrose Swasey and described in a little work by Prof. MacCord, called "The Teeth of Spur Wheels," published by Pratt & Whitney in 1881.

The same description may be found in the work on kinematics by the same author. This machine was designed to shape the templets used in the forming of rotary gear-cutters, and consequently does not strictly belong in the class of actual gear-generating machines which we are considering. The relative motions, however, of the tool and blank are necessarily the same as that of a gear-generating machine using the describing method to produce epicycloidal teeth. A small cylindrical cutter was used which was carried around first by a disc rolled on the outside periphery of a ring representing a pitch circle, and then by another disc moving as if it were rolling on the inside of the same circle, the transfer from one disc to the other being made automatically.

CHAPTER IV.

AMERICAN GENERATING MACHINES.

THE first machine involving the Sang theory—that is, using a rack-tooth shaped cutter as the generator and employing the intermeshing method to produce interchangeable gears—was the invention of Mr. Hugo Bilgram, of Philadelphia, and was patented in 1884. It marked a distinct advance in the art, for the reasons stated above, and also because it produced a new form of bevel-gear tooth, the octoid; the reason being, as has been said, that the tool, which represents a tooth of an involute crown-gear, is given the plane surfaces of an involute rack-tooth. A complete description of this machine appeared in the *American Machinist* for May 9, 1885, and, for the purposes of this article, the accom-

panying plan and sectional elevation, Fig. 23, will be sufficient to show the arrangement of parts and to make clear the operation. *K* represents the blank to which both motions of translation and rotation are given; that is to say, it is, in effect, rolled upon its conical pitch surface under and past the tool *T*, which is reciprocated by the ram *B* across the face of the blank. As the spaces between the teeth of a bevel-gear are tapering, one side of each tooth is finished first, the blank being indexed by the mechanism *LPN*. The tool can be adjusted laterally so as to bring the side at work into a radial plane of the pitch cone. The rolling motion is obtained by swinging the carrier *C*, on which the blank spindle *I* is adjustably supported in bearing *H*, around the vertical axis *YY*, in which lies also the apex of the pitch cone of the blank. This swinging movement causes a rotary movement of the blank on its axis by means of the two flexible steel-bands *Q* and *Q′*, which are

attached at one end to the frame and pass in opposite directions around the cone *M* on the blank spindle representing part of the pitch cone of the blank. The cones are reversed because they lie on opposite sides of the axis *YY*.

It will be convenient to refer here to Mr. Bilgram's improved bevel-gear generating machine, which was described in the *American Machinist* at page 114, vol. 25. The improvement consists in making the machine automatic, by providing indexing mechanism which turns the blank between each stroke of the cutter, and a positive cutter-lifter to clear the tool on its back stroke, during which the indexing takes place. Thus the tool makes one cut in each space of the blank, and then the blank is "rolled" slightly for the next series of cuts.

A later improvement patented by Mr. Bilgram early in 1904 consists in slightly curving the tangential path of the tool relatively to the blank, the object being to relieve the teeth at the points and

near the bases, whereby they run more quietly at high speeds.

Following the first Bilgram machine came one for generating spur-gear teeth, invented by Mr. Ambrose Swasey, the patent on which was granted in 1885, and which was described in the *American Machinist* for November 13, 1890. This machine also incorporated the Sang theory, but the application was quite different. Instead of one planing tool representing a crown-gear tooth, a gang of milling cutters, marked *K* in Fig. 24, are used, representing several teeth of a rack. The spur-gear blanks *G* are supported on a horizontal arbor, and the milling cutters pass tangentially above the periphery of the blank which travels at the same linear speed, the motion being exactly that of a gear and rack in mesh. A lateral feed was naturally required to cause the cutters to travel across the face of the blank, or blanks, as a number could be cut at once. The cutters present a novel feature, being divided in two parts along

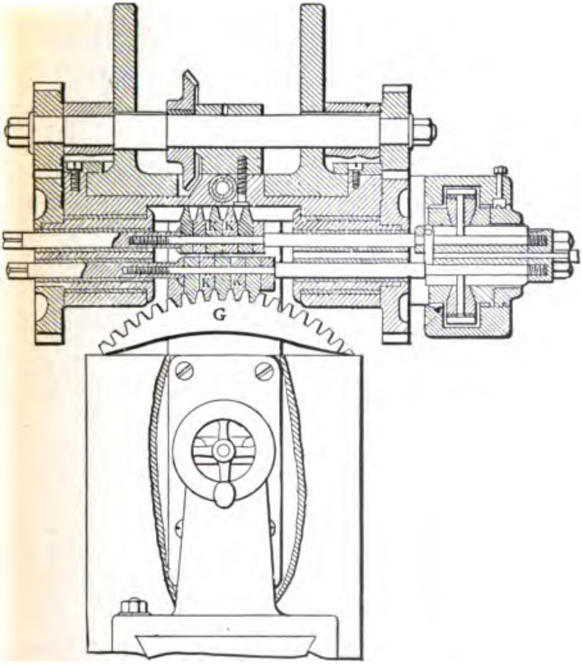


FIG. 24.—Swasey's Spur-gear Generator—1885.

an axial plane, and while the lower half engages the blank and travels forward with it, the upper half is moving backward, preparatory to engaging a fresh portion.

A machine working in the same way, but using a gang of solid cutters longer than the circumference of the pitch circle of the blank, was patented in 1896 by Mr. H. C. Warren, of Hartford.

There is but one American machine using the "describing" method known to the writer, and that was the invention of Mr. Grant, and was patented in 1889. The accompanying plan view and diagrams, Fig. 25, will make the operation clear. The tool is practically a point, although the one used for shaping the tooth-faces may have a straight cutting edge, which really makes that part of the operation an intermeshing one, as was the case in the Saxton apparatus hereinbefore described. Its path of reciprocation along the guide 38 represents an element of the cone 3 and the rotation

of the arm carrying that guide about the axis 33-1-18, together with the simul-

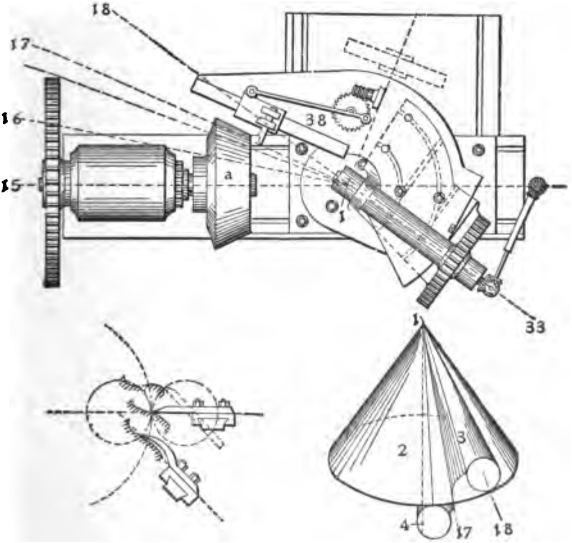


FIG. 25.—Grant's Bevel-gear Generator Using the Describing Method—1889.

taneous rotation of the blank, at angular velocities inversely proportional to the sines of the center angles of the pitch

cone and rolling cone, gives to the path of the tool relatively to the pitch cone of the blank the motion that an element of the rolling-cone 3 would have relatively to cone 2. Consequently, the point of the tool describes an epicycloidal surface on the face of the tooth, the blank having been preliminarily grooved so that the tool has only to remove a small amount of metal. In the plan view the center angle of the pitch cone is represented by the dotted lines 15–1–17, and that of the rolling cone by the lines 17–1–18. To shape the hypocycloidal flanks the curved tool shown is used, and the tool-slide carrying arm is adjusted so that the center angle of the rolling cone becomes the angle indicated by dotted lines 17–1–16, the axis 33–1–18, about which the arm rotates, having been adjusted into the line 16–1. The path of movement of the tool now becomes in effect that of an element of the cone 4 rolling on the inside surface of cone 2. As has been said, this is the only machine

of American invention, as far as known to the writer, working on the describing method, the others employing it being foreign inventions.

Another machine invented by Mr. Grant works on the intermeshing method to produce "planoid" teeth on bevel-gears, a name coined for them by the inventor. This machine was described in the *American Machinist* for June 7, 1894. The "planoid" bevel-gear tooth has plane flanks, preferably radial, but not necessarily so, and faces which are conjugate to the plane flanks of the mating gear. They closely resemble epicycloidal faces, and as the angle of the bevel-gear cone decreases the resemblance increases, until in spur-gears planoid teeth become epicycloids. The operation of the machine is similar in some respects to that of the Saxton device described above, in that the tool used represents the plane flank of a mating gear, and will be understood from the sectional elevation and diagrammatic views, Fig. 26. The tool

t^2 is reciprocated along the guideway shown, and is first fed in radially to the full depth of a tooth by means of the worm and sector, the blank being meanwhile held stationary; it is then withdrawn as far as the pitch circle, and is kept there, continuing to reciprocate, of course, while the blank is rolled out of engagement with it, as if it were rolling on the pitch surface *c* of the gear whose tooth flank the tool represents, by turning the sprocket wheel *m* which rotates the blank and also swings the carrier *B* about the axis 3–4.

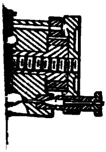
Mr. H. C. Tyler's 1895 machine, from which the views of Fig. 11 were taken, was arranged like a vertical slotter (see Fig 28), 9 being the slotter bar and 41 the tool of involute rack shape. The standard *C* on which the bar 9 reciprocates is fed across the bed by a feed screw 22, driven by the worm-gearing and ratchet mechanism shown, from the driving pulley 7. The blank, shown at *T* as a templet, is held on a swinging arm *S*,

the front curved end of which is attached by two flexible steel bands running in opposite directions to the movable standard *C*. Thus while the tool is relatively rapidly traveling up and down it is fed tangentially to the blank, which is simultaneously rotated at the same linear speed, the motion of course being that of a rack and gear in mesh.

CHAPTER V.

MODERN GENERATING MACHINE.

THERE is one modern machine using the describing method, but shaping the teeth to the approximate circular arcs, which should be mentioned, and that is the machine built by Smith & Coventry, of Manchester, England, which was described in the editorial letters from the Paris Exposition, at page 951, Vol. 23. In this machine the relative movement of the cutters and blank—for two planing tools are used, operating on opposite sides of the same tooth—is compounded from a swinging movement of each in planes at right angles to each other. The tools are reciprocated along guideways pivoted at the cone apex of the blank, the guideways being separated at a rate controlled by the swinging of the blank-supporting



carrier in a horizontal plane about a vertical axis through the cone apex.

The Fellows gear-shaper, described in the *American Machinist* at pages 519, Vol. 20, and 153, Vol. 23, works on the intermeshing method, using a complete gear instead of a rack-tooth as the generator. Back of it, however, is a machine for finishing the hardened generating gear cutter, which uses a grinding disc, representing one side of a tooth of an involute rack, or crown gear. The grinding machine will be described afterwards. Presumably most of the readers of the *American Machinist* are familiar with the operation of the gear-shaper. The gear *C*, Fig. 29, which is transformed into a planing tool by giving back and side clearance and top rake to the teeth, is reciprocated vertically and works with a draw stroke, except on internal gears, when it has to plane downwards. It and the blank, a number being shown at *b*, are slowly rotated as if in mesh. The addenda of the cutting teeth are increased,

so that one tool can cut small and large gears and a rack of the same pitch, which will all mesh perfectly together. The same tool can be used for internal gears also, which can be cut on this machine as easily as external toothed wheels, but bevel-gears cannot be cut. If the path of travel of the generating-gear is not quite parallel to the axis of the blank, it will produce a tapered gear larger at one end than at the other, which is not a true bevel-gear, since the teeth are the same depth and have the same side curves throughout their length. A photograph of this abnormal gear is shown in the *American Machinist* at page 177, Vol. 23.

Used as one of the generating-cutters, it has the advantage that as the face is ground away to sharpen it the curvature of the teeth does not change, as it would in an ordinary bevel-gear. To make one of the generating-gears, a blank is put in the machine and another generating-gear set to work upon it, a striking

instance of what may be called, for want of a better term, "self-reproduction," which pervades all machine-tool work. The gear thus made, which is left a little large, is hardened and then put in the finishing machine (see Fig. 30), which grinds the teeth to true involute or octoid shape, the machine being also adapted to finish hardened bevel-gears. One side, 2, of the emery wheel d, which is a plane surface, represents, when acting on a tooth side of the blank c, one side of an involute rack-tooth, and the blank is rolled in mesh with it by means of the handle b^5, the movement of translation being caused by the two flexible steel bands e and e^1. If the blank is large the emery wheel may be reciprocated across its face by means of the rack d^5, pinion d^6 and handle d^7, the same means serving to withdraw it from engagement during the indexing of the blank. In order to give the active surface of the grinding disc the angularity relative to the blank which the side of the rack-tooth would have, the

carrier for the blank is arranged at an angle to the horizontal, as shown.

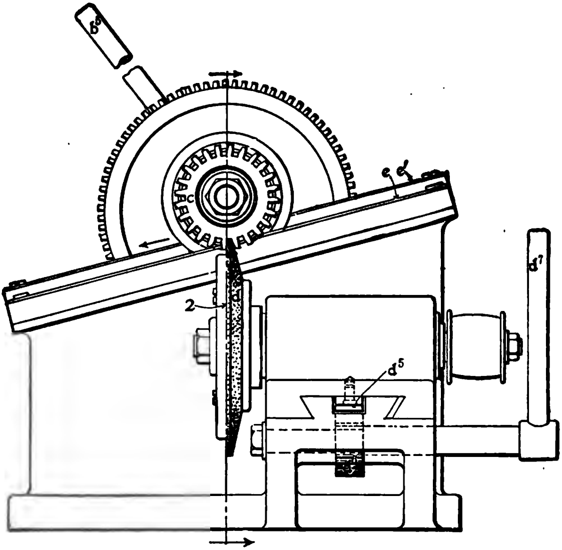


FIG. 30.—The Fellows Cutter and Bevel-gear Generator.

This hand-operated machine serves to illustrate the operation upon which an automatic machine invented by Mr. Fellows works.

A rack-generating machine working on the principle of the gear-shaper was also invented by Mr. Fellows, in which both motions of rotation and translation are given to the generating-gear, the rack being held stationary except for a back and forth movement to clear it from the cutter on the back stroke. Spiral gears can also be cut on the shaper by giving the generating-gear an additional rotary movement as it planes across the face of the blank.

The Pope Company, of Hartford, had at work at one time a large number of machines of the intermeshing Sang theory type, making bevel-gears for chainless bicycles, the inventor of which was Mr. H. C. Warren, and which were described in the *American Machinist*, at page 211, Vol. 21. In these machines two rotary cutters are used mounted on axes inclined towards each other, the peripheries being beveled off on adjacent sides, the beveled portions representing, where they engage the blank, opposite sides of adjacent teeth

of an imaginary crown-gear. The rolling or intermeshing movement is divided up between the blank and the cutters; that is, the blank rotates on its axis, and an oscillating carrier on which the cutter carriages are slidably mounted is swung, at a proper velocity ratio, around the axis of the imaginary crown-gear, thus giving to the active portions of the cutters and to the blank the relative movements of the crown-gear teeth and the bevel-gear teeth in mesh. Fig. 31 serves to illustrate this action, although the cutters therein shown are broaching or filing tools and do not rotate, nor do they need in that case any feeding movement, whereas when rotary cutters are used, they must be gradually fed across the face of the blank, since the active surfaces, being conical, only touch the sides of the bevel-gear teeth in a point and not in a line, as would an actual crown-gear tooth.

The machine invented by Mr. O. J. Beale, and described in the *American Machinist* at page 272, Vol. 22, forms,

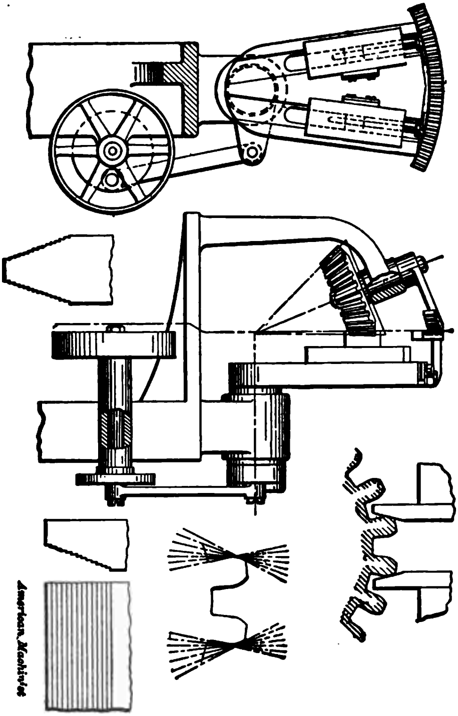
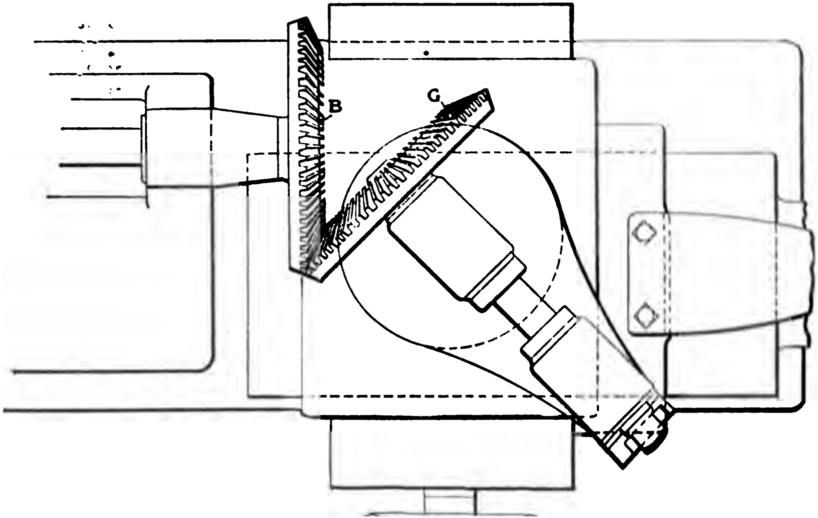


FIG. 31.—Principle of the Warren Bevel-gear Generator—1885.

with the Ingold device hereinbefore mentioned, the nearest practical embodiment of the molding process. The generator is a hardened gear, usually a crown-gear, and is run in mesh with the bevel-gear being formed, requiring no additional cutting movement. This generating-gear is formed by cutting down the teeth of a crown-gear having straight sides to varying hights, leaving sharp edges. The generator, as at *G*, Fig. 32, where it is shown as a complete crown-gear used to burnish and compress the teeth of the blank *B* after the generator *C* has been used on it, is brought up into mesh with the gear to be finished, the teeth of which have been previously formed to approximately correct shape, and is rotated and gradually forced forward until the teeth of the gear are all correctly shaped.

Better results are obtained if the direction of rotation of the mating generator and gear is intermittently reversed.

The descriptions of the modern machines just given and those which follow


B
G

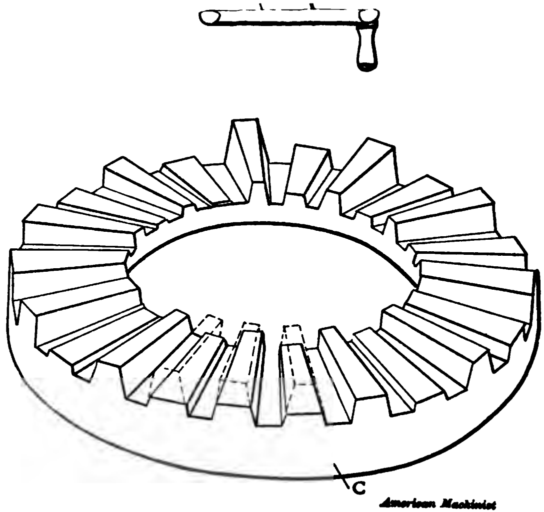


Fig. 32.—The Beale Bevel-gear Generator—1899.

are not arranged according to a strict chronological order, but rather grouped according to similarity of operation.

The machine brought out by the Leland & Faulconer Company, which grinds the teeth of a hardened bevel-gear to the form of a templet, was described in this paper at page 589, Vol. 22, but the generating machine of the same company invented by Mr. Cheney has not received notice, as far as the writer is aware. It is also for the purpose of finishing hardened bevel-gears and uses an emery wheel the periphery of which has the cross-section of the involute rack-tooth. It has besides the rotary motion a reciprocatory one across the face of the blank. The blank has both movements which go to make the necessary rolling motion, being mounted on a swinging carrier and also rotated on its axis. One side of each tooth is finished first, as in the Bilgram machines described.

Among bevel - gear planers which govern the relative movement of the tool

and blank by a templet, the machines of the Gleason Tool Company, are prominent. Generating machines are employed in the production of the templets, which were described in the *American Machinist* at page 1058, Vol. 23. A bevel-gear generating machine was patented by Mr. James E. Gleason in 1898, which I understand has not been placed on the market owing to the fact that the performance of the templet machines is so satisfactory. The distinguishing characteristic in the operation of the generating machine is that the rotary cutter finishes both sides of a space in one traverse across the face of the blank. This is accomplished by giving it a lateral reciprocatory movement in addition to its other motions, equal in amount to its width at the periphery, so that first one side and then the other is brought into the vertical plane through the axis around which the blank may be considered as rolling. Each time the cutter is moved laterally the direction of

oscillation of the blank and of the cutter carrier is changed. In this machine the rolling or intermeshing movement is obtained by a rotation of the blank and a swinging of the cutter carrier about the vertical axis which passes through the cone apex of the blank. The lateral movement of the cutter and the reversal of the intermeshing motion take place after every two revolutions of the cutter.

It was intimated near the commencement of this article that a hard and fast line cannot be drawn between the different types of gear-cutting machines. This is illustrated by a templet bevel-gear machine described in a British patent of 1850 granted to an engineer named John Hunt, and also very clearly by the machine invented by Mr. C. D. Rice, of Hartford, which was described in this paper at page 440, Vol. 23, in which the blank is given a rolling motion relative to the active face of the cutter to cause it to shape the tooth-face. The photograph of the model, illustrating the principle,

which accompanied the article in the number referred to above, is reproduced here in Fig. 33. A swinging motion is

FIG. 33.—Principle of the Rice Machine.

given to the carrier, on which the blank and master gear are mounted, about the vertical axis through the cone apex, which motion causes a rotation of the blank by engagement of a tooth of the master gear

with the guide plate, the active face of which lies in the same plane as the active face of the cutter. This is a true intermeshing movement, and this machine consequently involves the intermeshing method, although the tooth curves are determined by a master gear or templet.

The most recent bevel-gear generator of the intermeshing "Sang theory" type is that of Mr. Beale, of the Brown & Sharpe Manufacturing Company, which was described in the *American Machinist* for January 20, 1903. The distinguishing feature of this machine, aside from any consideration of the details of the mechanism, lies in the fact that both sides of a space are finished simultaneously. Two toothed discs are used, mounted on axes inclined to each other, the teeth of one occupying the spaces of the other. The outer faces of the discs are planes in which the cutting edges lie, and represent, in those portions of them which engage the blank, the two sides of an involute crown-gear tooth, the same assumption being

made, as in most machines of this type, that the crown-gear teeth have plane sides like the rack-teeth. The axes of the cutters are stationary in operation, both motions of rotation and translation being given to the blank. The cutter spindles are mounted on a sliding head which is fed up a trifle for the backward swing of the blank so as to let the cutters take a finishing cut. Adjustments of the cutter spindles are provided so as to permit the formation of gears of various diameters, pitch and angles of pressure.

The somewhat intricate but very ingenious machine invented by M. Monneret, of Paris, which was described in the *American Machinist* at page 683, Vol. 23, generates helical bevel-gear teeth by the intermeshing "Sang theory" method. Briefly stated, the operation of this machine consists in reciprocating the tool in a plane representing the side of a crown-gear tooth, the guideway on which the tool-holder slides being pivoted on an axis coinciding with that of the imaginary

crown-gear. The blank is rotated at the same time, so that the tool cuts a helical groove, and this rotation being continuous serves also to space the blank for the next cut. After one cut has been made in each space all around the blank, the tool is fed in by swinging its guide slightly, the rotation of the blank being simultaneously modified by adding to it the angular movement corresponding to the intermeshing movement of a tooth with the crown-gear tooth represented by the tool. It will be seen that, for gears of the same pitch, the rotary movement to cause the helical cut, which is quite small in practice, takes care of itself, so that the operator does not have to think about that at all.

CHAPTER VI.

GENERATING OTHER FORMS OF TEETH.

GOING back now ten years or more, we find a machine for generating the teeth of spiral gears forming the subject of an 1889 patent to E. P. and H. C. Walter, of Bridgeport, Conn., one view of which is shown in Fig. 34. This machine serves to illustrate the similarity spoken of above between the intermeshing method of gear generating and the process of cutting a helix, or, as it is generally called, a thread, in the lathe. The tool *O* is of truncated wedge shape—that is, of the form of an involute rack-tooth—and is reciprocated by the screw and reversing pulleys shown, and has no other movement. By turning the crank *X* which is fast on a feed screw lying directly under the blank arbor and engaging a nut on the

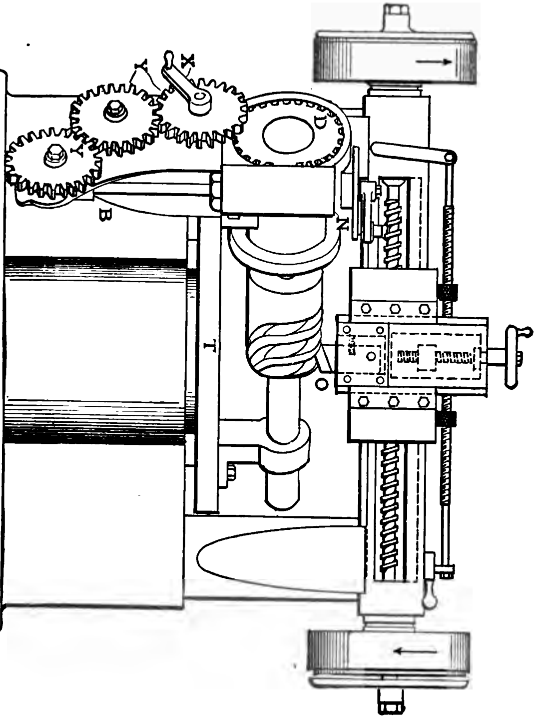


FIG. 34.—Walter's Spiral Gear Generating Machine—1889.

carriage T, the blank is fed along in the direction of its axis. It is also simultaneously rotated by means of the change gears Y, worm shaft B and worm wheel D. The index mechanism N provides for the cutting of multiple threads. At first glance it seems that there ought to be a fixed relation between the reciprocations of the tool and the movements of the blank, but a little study shows why it is unnecessary and also shows that the operation, although somewhat disguised, is essentially the same as that of cutting a thread in the lathe, the tool being given an additional cutting movement.

It will readily be seen that the same relative action takes place whether the rotating work is moved axially past the tool or whether the tool is moved along parallel to the axis of the rotating work. The latter is the ordinary lathe arrangement, while the former is that of this machine and would be that of a lathe if the tool carriage were clamped to the bed

and the headstock and tailstock were connected together and fed along the ways, as is the case sometimes in grinding machines.

In a screw-cutting lathe the change gears which determine the pitch of the thread being cut, fix a ratio between the rotations of the work and the travel of the tool parallel to the axis of the work. In the Walter machine, as the blank does the travelling instead of the tool, the change gears determine the ratio between the rotation of the blank and its travel in an axial direction.

If a very coarse pitch thread were cut in a lathe, the lead screw would have to perform too much of the work of cutting, since the rotation of the work must be slow compared to the feed of the tool. This is avoided by giving the tool an additional cutting movement, which is usually rotary but might be reciprocatory, as in the case under consideration. The point of the ordinary thread tool, if it had an additional reciprocating cutting move-

ment, would trace a straight line perpendicular to the axis of the work, whether the tool were set "square" with the work or obliquely. This is not so in the Walter machine, and a little consideration shows why. Supposing the pitch of the thread to be cut by the ordinary thread tool were increased until it was infinite, then the rotary movement of the work would be zero and the cutting movement would be caused entirely by the lead screw. It is evident that the tool would have to be arranged with its cutting face in a vertical plane instead of in a horizontal plane, as ordinarily. So, if the lead of the screw being cut were 45 degrees, the face of the tool should stand at an inclination of 45 degrees to the horizontal. For ordinary pitched screws cut by a thread tool the inclination would be very slight and is unnecessary, but if the tool has a reciprocatory cutting movement and the pitch of the thread is coarse, as is the case in Walter's machine, the path of movement of the tool must

correspond with the lead of the thread, otherwise there would be trouble at once, owing to the tool not tracking in the groove that it cut. This angular adjustment, it is noticed, is provided for in the thread-milling machine of Pratt & Whitney described in the *American Machinist* of November 6, 1902. It will now be seen why it is that the reciprocatory movement of the tool in this machine is entirely independent of the movements of the work, and that the machine in its operation is similar to a screw-cutting lathe, or thread-milling machine.

Since the use of a tool having the shape of an involute rack tooth in this machine causes the resulting teeth of the spiral gear to be conjugate to it, it is at once evident that the same result must be obtained when the pitch is finer; in other words, that screw-threading machines and all other machines which produce helical gearing of any kind, work on the intermeshing generating principle, as stated near the commencement of this

treatise, where worm and worm-wheel cutting machines were cited as examples.

It must be borne in mind that there is no dividing line between an ordinary spur gear and an ordinary screw thread, both being forms of toothed gearing. A worm, which is generally considered to be a gear, is nothing but a screw thread, single or multiple, and the teeth of what are ordinarily known as spiral gears are merely short sections of screw threads of coarse pitch.

The third and last group of recent inventions which remains to be considered comprises five machines of the intermeshing type which were especially designed to generate special forms of bevel-gears for chainless bicycles, although of course adapted for other uses. Each of them forms a gear to mesh with a "pin-wheel" and uses a cutter or cutters of the form of the pins, which are usually cylindrical, but may be conical or of other suitable form, giving to the blank and cutter exactly the motion that

the finished gear and pin-wheel have when in operative engagement. The axes of the gear and pin-wheel may or may not be at right angles to each other, and some of the machines provide adjustments for the cutter head to provide for the cutting of gears having different angularity of axis relative to that of the pin-wheel.

The principal difference in the operation of these machines, aside from the details of mechanism, which naturally vary considerably, lies in the additional cutting movement imparted to the cutters, and as three of them are similar in this respect, a description of the operation of one of them will suffice. The machine shown in Fig. 35, invented by Mr. E. G. Ashley, of Rochester, N. Y., to cut what is known as the "Sager gear," is taken as an example of this type. At *X* is the blank which has been previously gashed out with the square slots shown, and *H* indicates the milling cutters representing the pins of the pin-wheel with

which the finished gear is to mesh. If as many cutters are used as there are pins on the wheel, every tooth will be finished

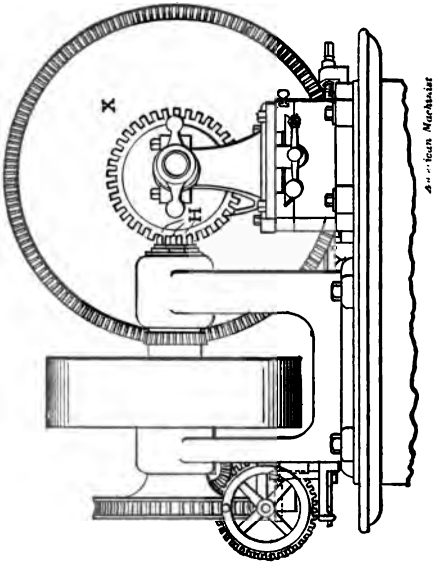


FIG. 35.—Ashley's Machine for Generating Sager Gears.

in one revolution of the blank, but in this instance only one-half as many cutters as pins are used, so that two revolu-

tions of the blank are necessary to complete it. One cutter only might be used, which is the case in the two other machines of this type; but then the blank would have to be indexed. In each of the machines means are provided to slowly revolve the rotating cutter, or cutters, about an axis corresponding to that of the pin-wheel, and to simultaneously rotate the blank at the proper velocity ratio.

The other two patents for machines using rotary milling cutters of the exact form of the pins of the wheel were granted to Mr. H. F. Cuntz and Mr. J. S. Copeland, and were both assigned to the Pope Company, now the American Bicycle Company.

These gentlemen were also the inventors of the two other machines in which different kinds of cutters are used; in one of which—that of Mr. Copeland (see Fig. 36)—the spaces in the blank *p* are planed out by the tools *g*, which in number, form and arrangement resemble the

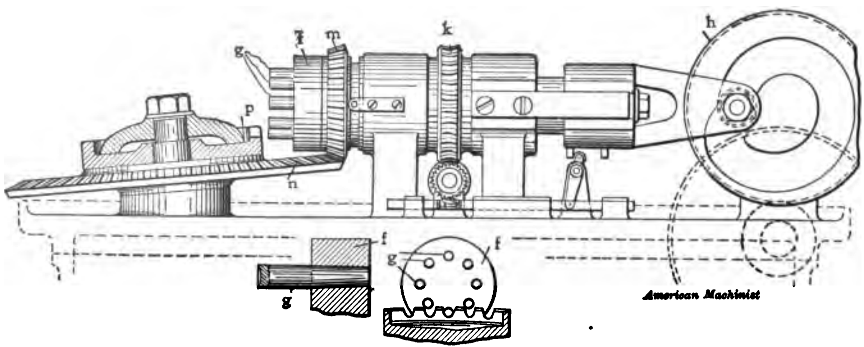


Fig. 36.—Copeland's Generator for Bevel-gears to Mesh with Pin Wheels.

pins of the wheel with which the gear being cut is to mesh. The head *f*, in which the cutters are mounted, is reciprocated by the cam *h* to carry the cutters across the face of the blank, and is rotated by the worm and worm-wheel *k*, the blank being rotated in unison by the bevel gears *m* and *n*. The small views show one of the cutters detached and an end view of the cutters in engagement with the blank.

The cutter head of the other machine invented by Mr. Cuntz is illustrated in Fig. 37, from which it will be seen that rotary toothed disc cutters are used, a cross-section of the periphery of which is that of one-half the pins of the pinwheel. The cutters represent the pins in number, location and movement relative to the blank.

This completes the list of gear-generating machines invented up to the present time, as far as known to the writer. The question naturally presents itself, after reviewing a class of machines as has here

been done, as to the lines along which development has occurred, and as to those along which it may be expected to occur in future. It appears safe to say

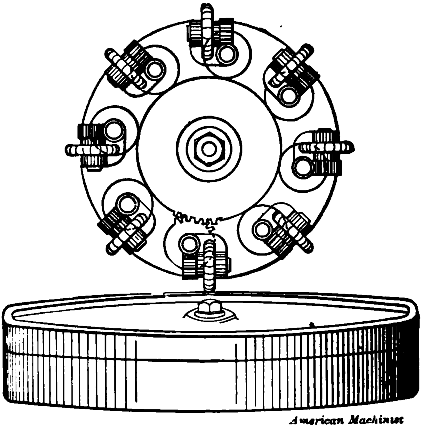


Fig. 37.—Cuntz's Generator for Bevel-gears to Mesh with Pin Wheels.

that the describing method has been superseded by the intermeshing method, and that for the production of ordinary form of gears—*i.e.*, spur and bevel-gears

—in interchangeable sets, the Sang theory and intermeshing method are indispensable.

In concluding, the writer wishes to acknowledge his indebtedness to Francis H. Richards, Esq., of New York, for valuable assistance regarding the data of the publications containing the descriptions of generating machines.

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